# Undecidability of Coverability and Boundedness for Timed-Arc Petri Nets with Invariants

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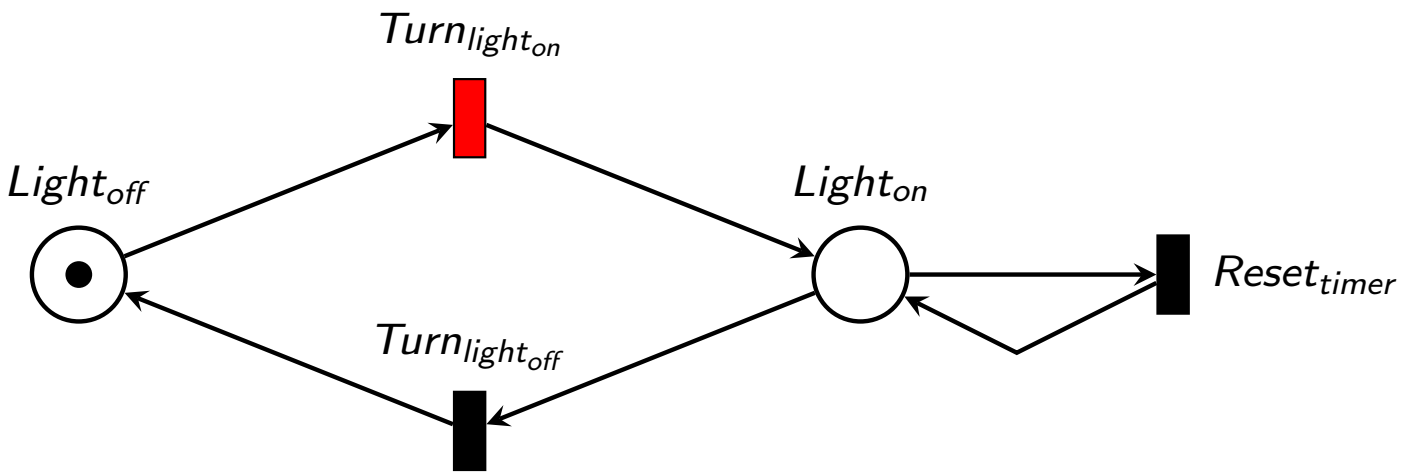
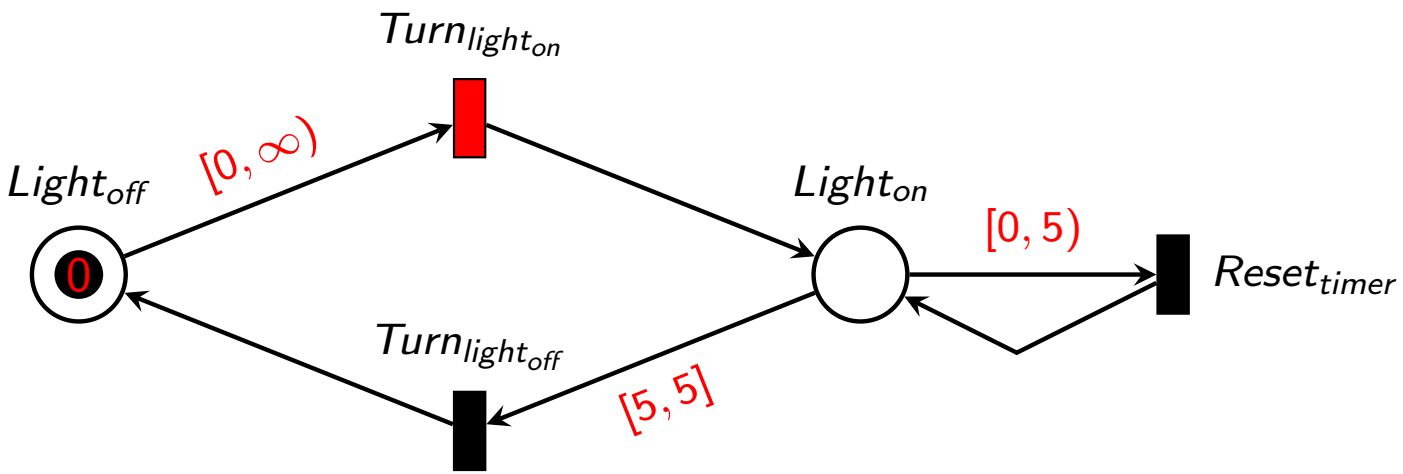
14. november 2009

# Example: Petri Net




$M_0$

# Example: Timed-Arc Petri Net

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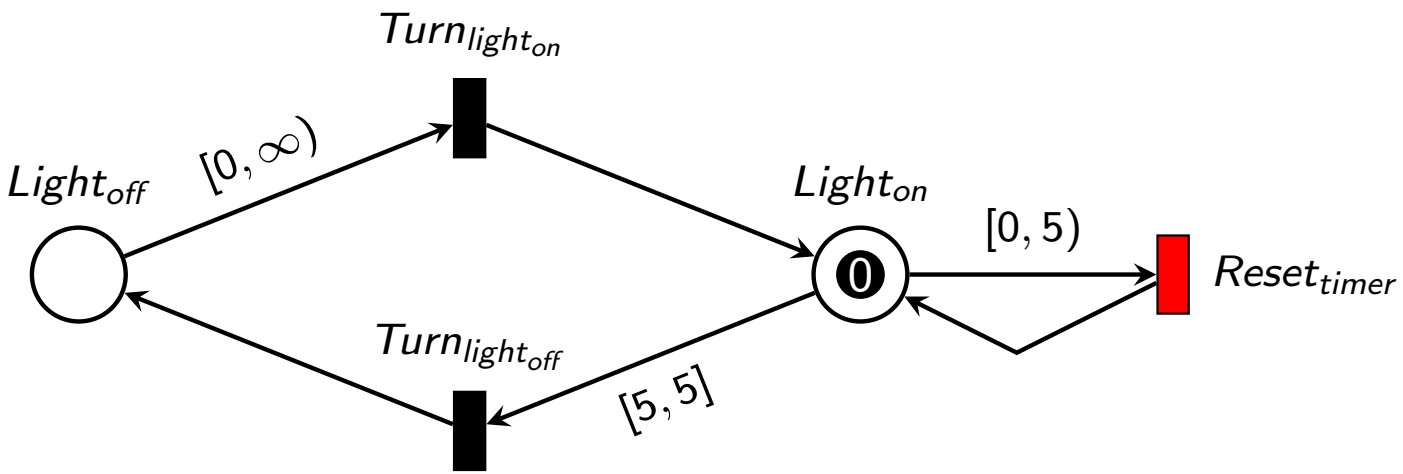


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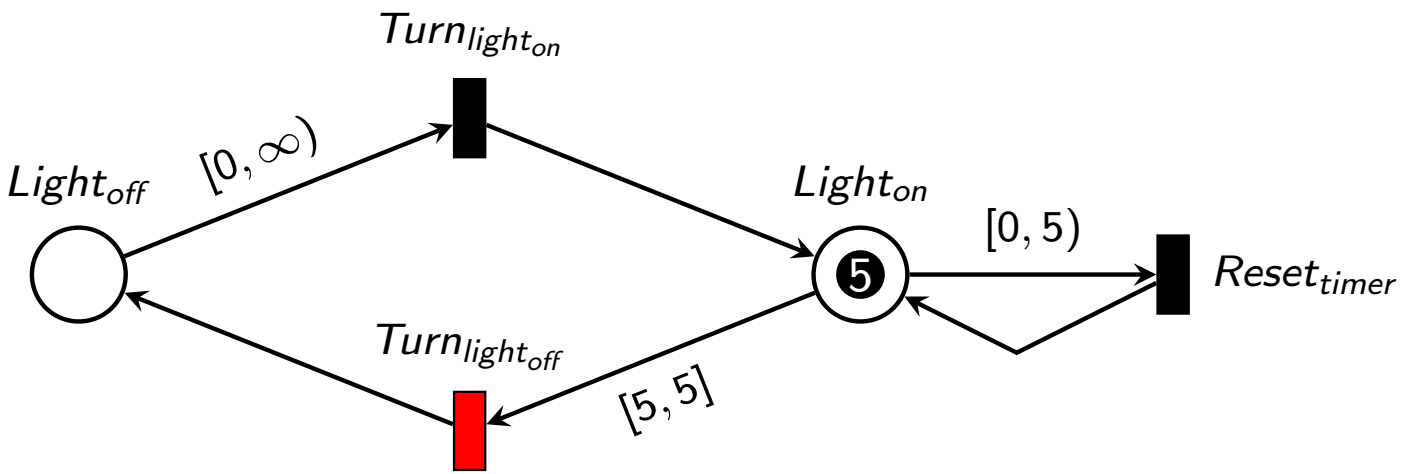


$$M_0 \xrightarrow{Turn_{light_{on}}} M_1 \xrightarrow{d:5} M_2$$

# Overview

Timed-Arc Petri Net was first introduced by

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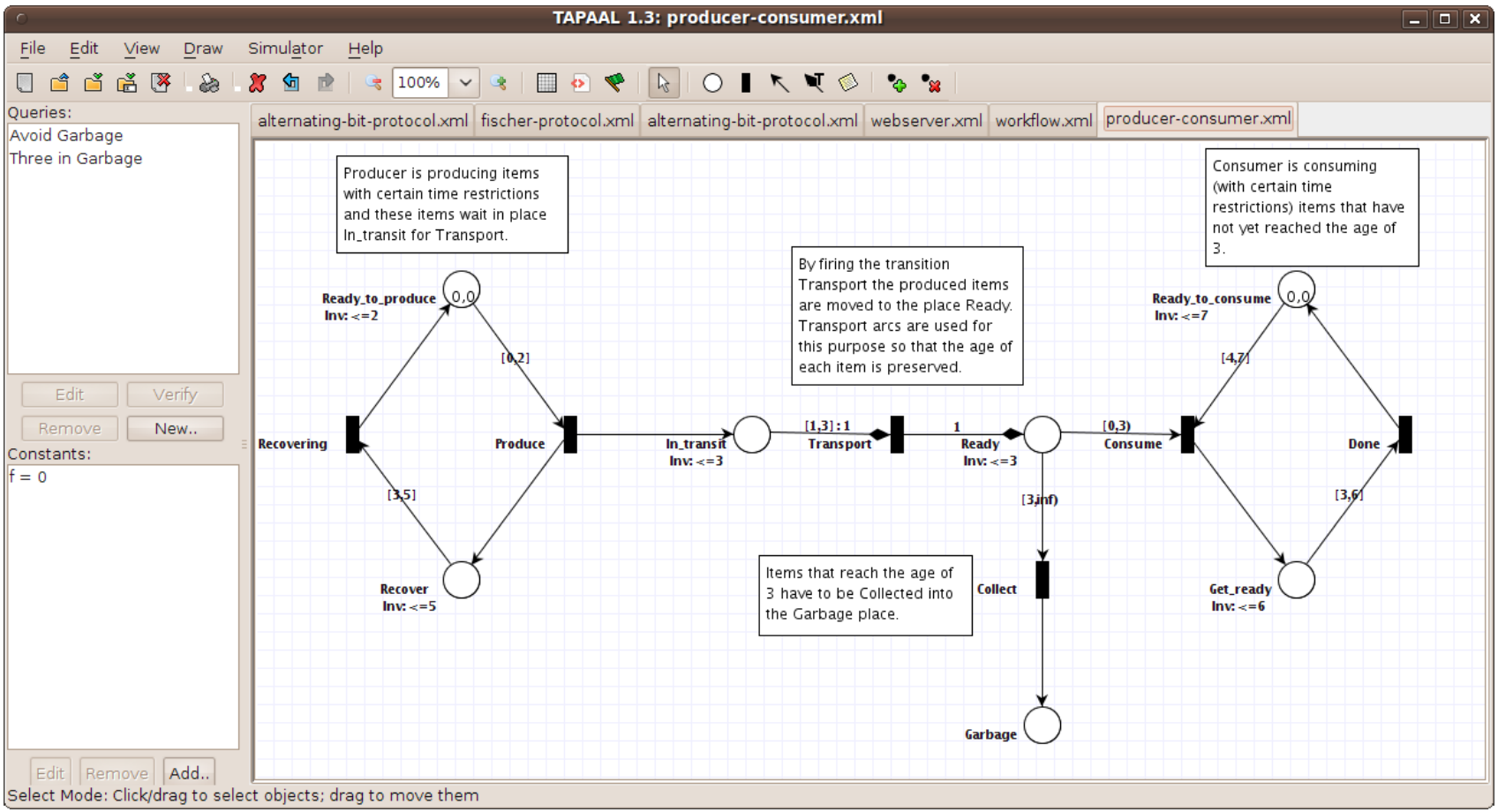
# Introduction

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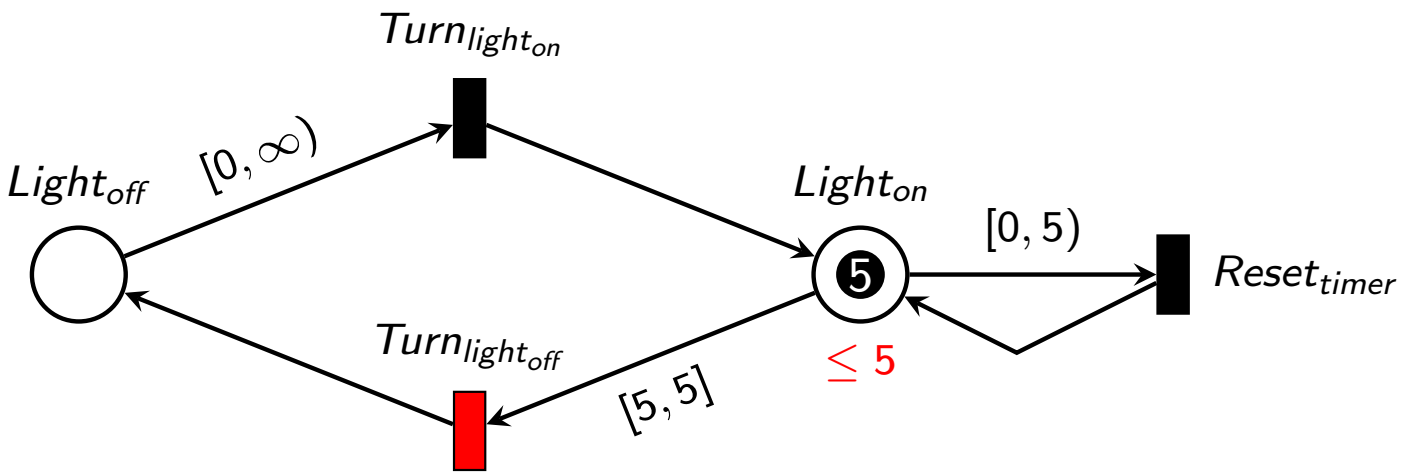


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# Boundedness and Coverability

## Boundedness

A marked ITAPN $(N, M_0)$ is said to be *bounded*, if there exist a number $k$ such that the number of tokens at any place in any reachable marking is bounded by $k$.

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## Coverability

A marking $M$ is said to be *coverable* from $M_0$ if there exists a reachable marking $M'$ which covers $M$.

# Our results

- Boundedness is undecidable for ITAPN
- Coverability is undecidable for ITAPN.

The idea is to make a reduction from a Minsky Two-counter Machine.

## Definition: Minsky Two-counter Machine

A Minsky Two-counter Machine (2-CM) with two non-negative registers $r_1$ and $r_2$, is a sequence of instructions,

$$\begin{aligned} I_1 &: Ins_1 \\ I_2 &: Ins_2 \\ &\vdots \\ I_e &: HALT \end{aligned}$$

where each instruction $Ins_j$ is one of the two types:

Inc $r_i := r_i + 1$; goto $I_k$;
where $i \in \{1, 2\}$ and $k \in \{1, 2, \ldots, e\}$.

TD if $r_i > 0$ then $r_i := r_i - 1$; goto $I_k$; else goto $I_\ell$;
where $i \in \{1, 2\}$ and $k, \ell \in \{1, 2, \ldots, e\}$.

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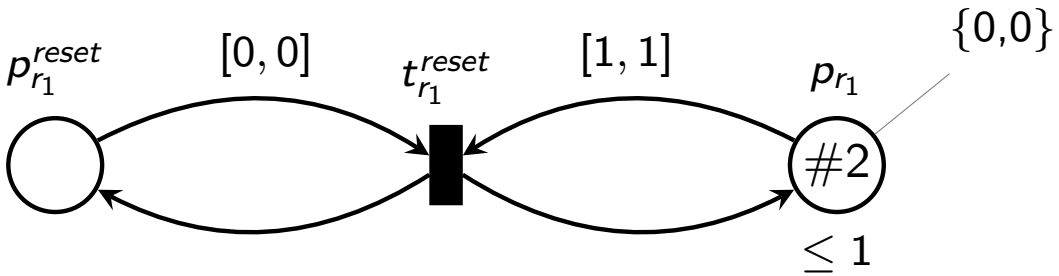
- A place $p_j$ for each instruction, simulating which instruction is about to be executed.
- A place $p_{count}$ to count the number of instructions performed.

# Simulation of the registers and the Halt instruction

Simulation of register $r_1$ with value 2.



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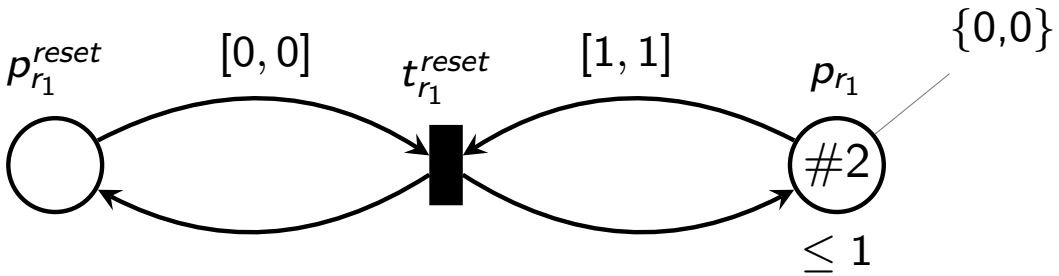
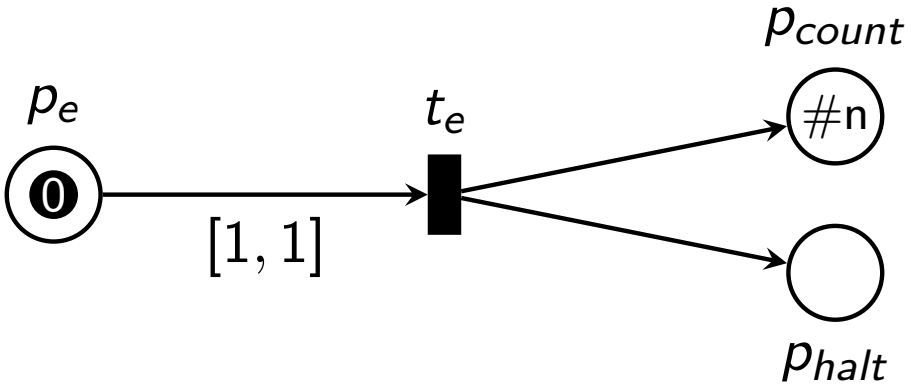



The HALT instruction is simulated by:



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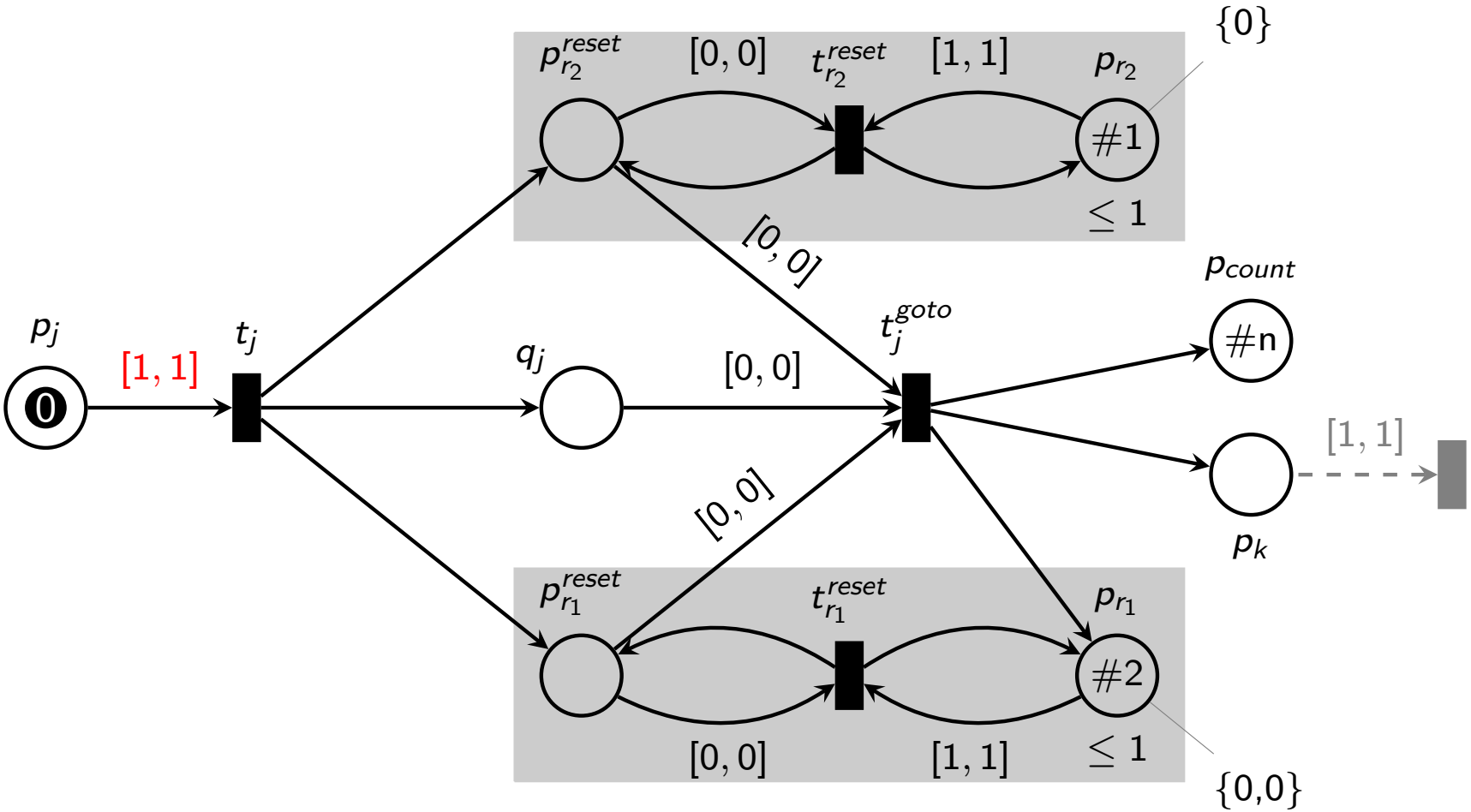

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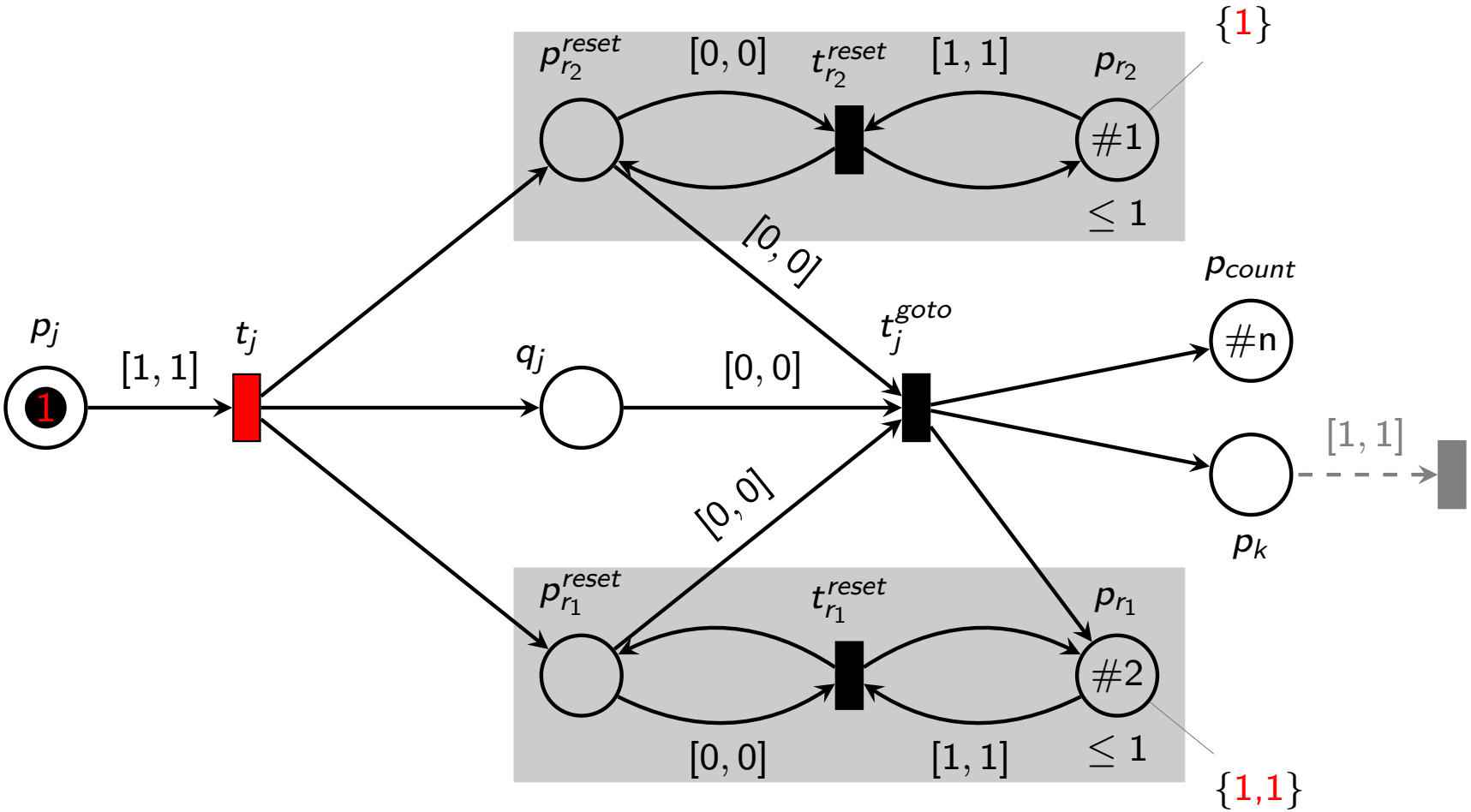

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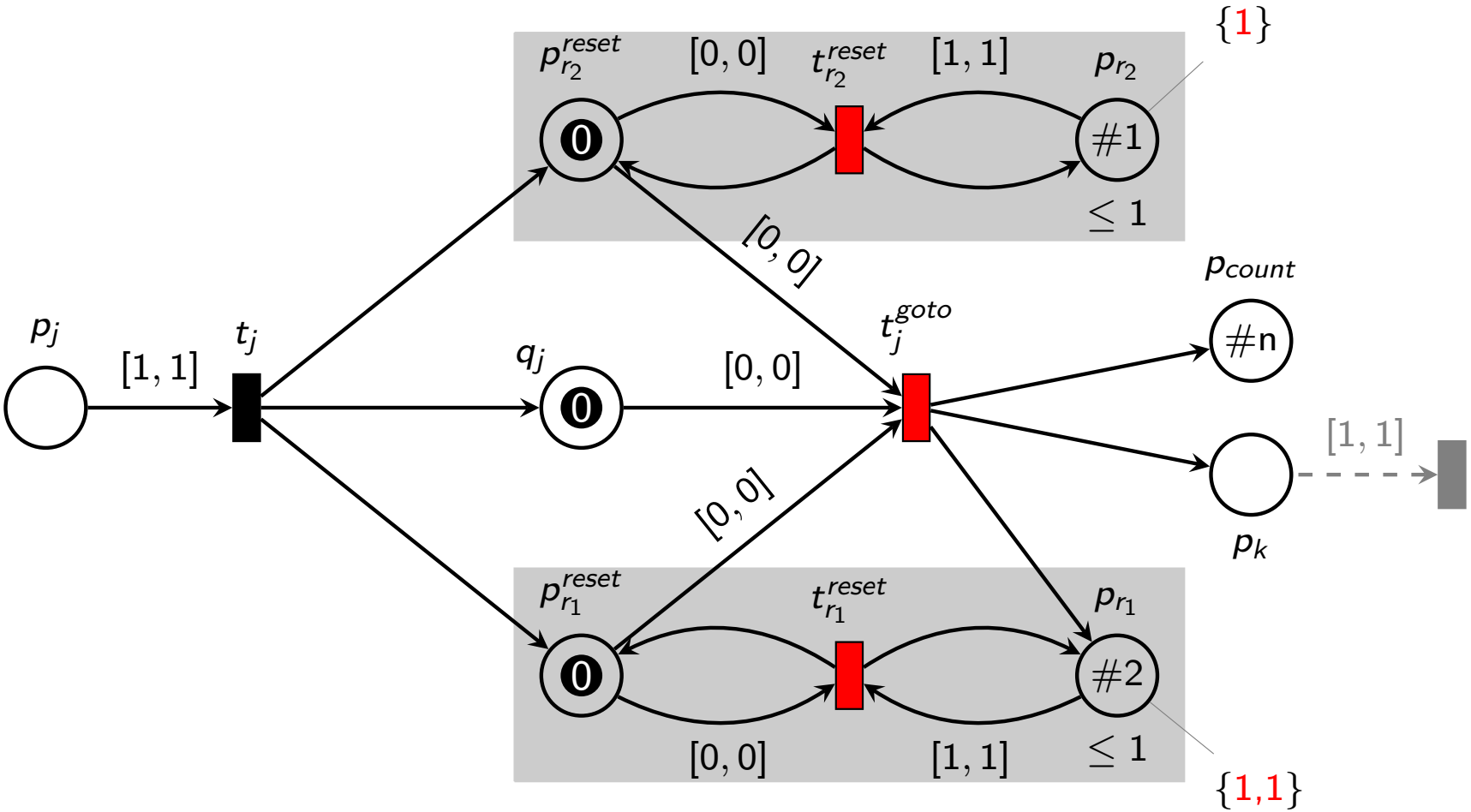

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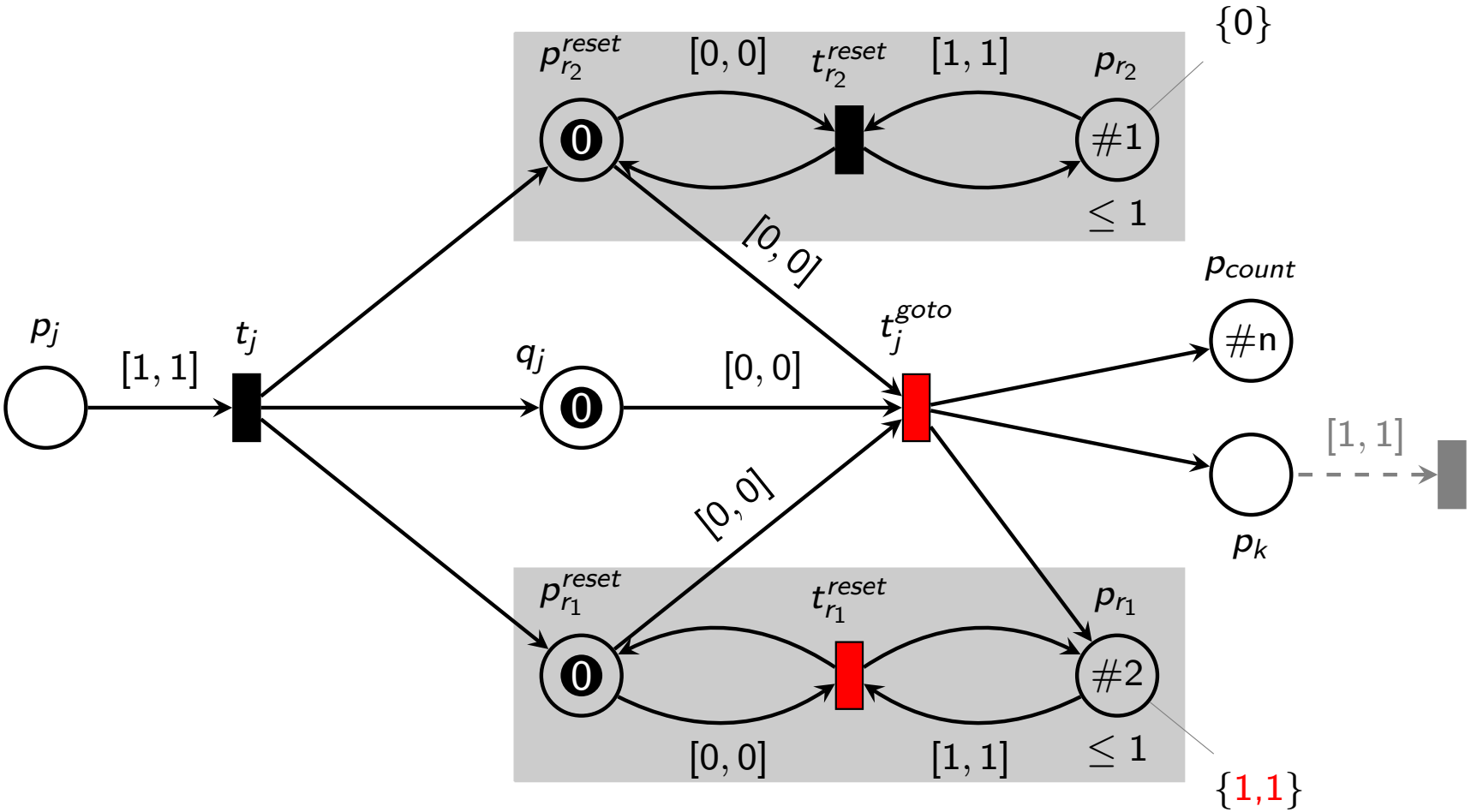

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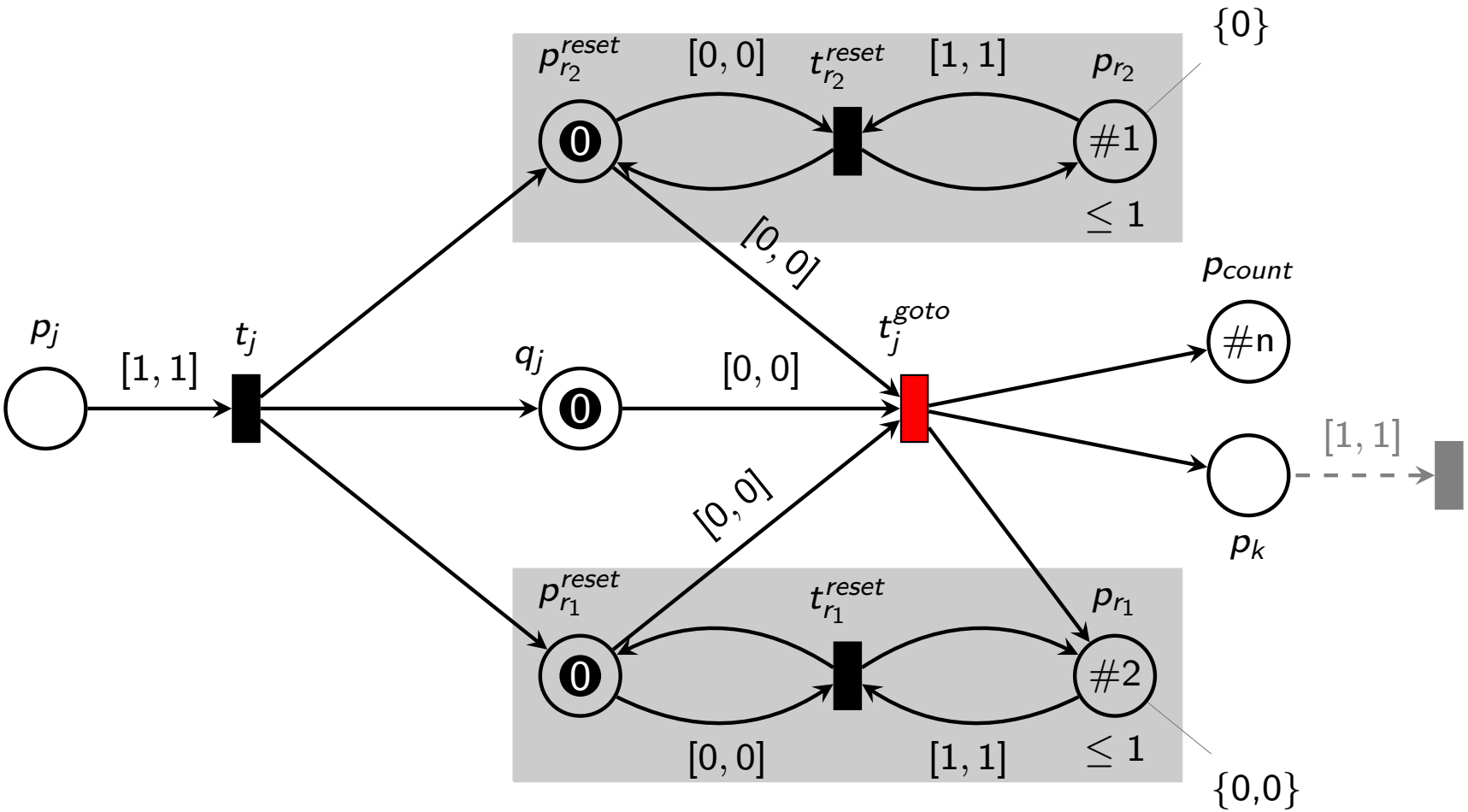

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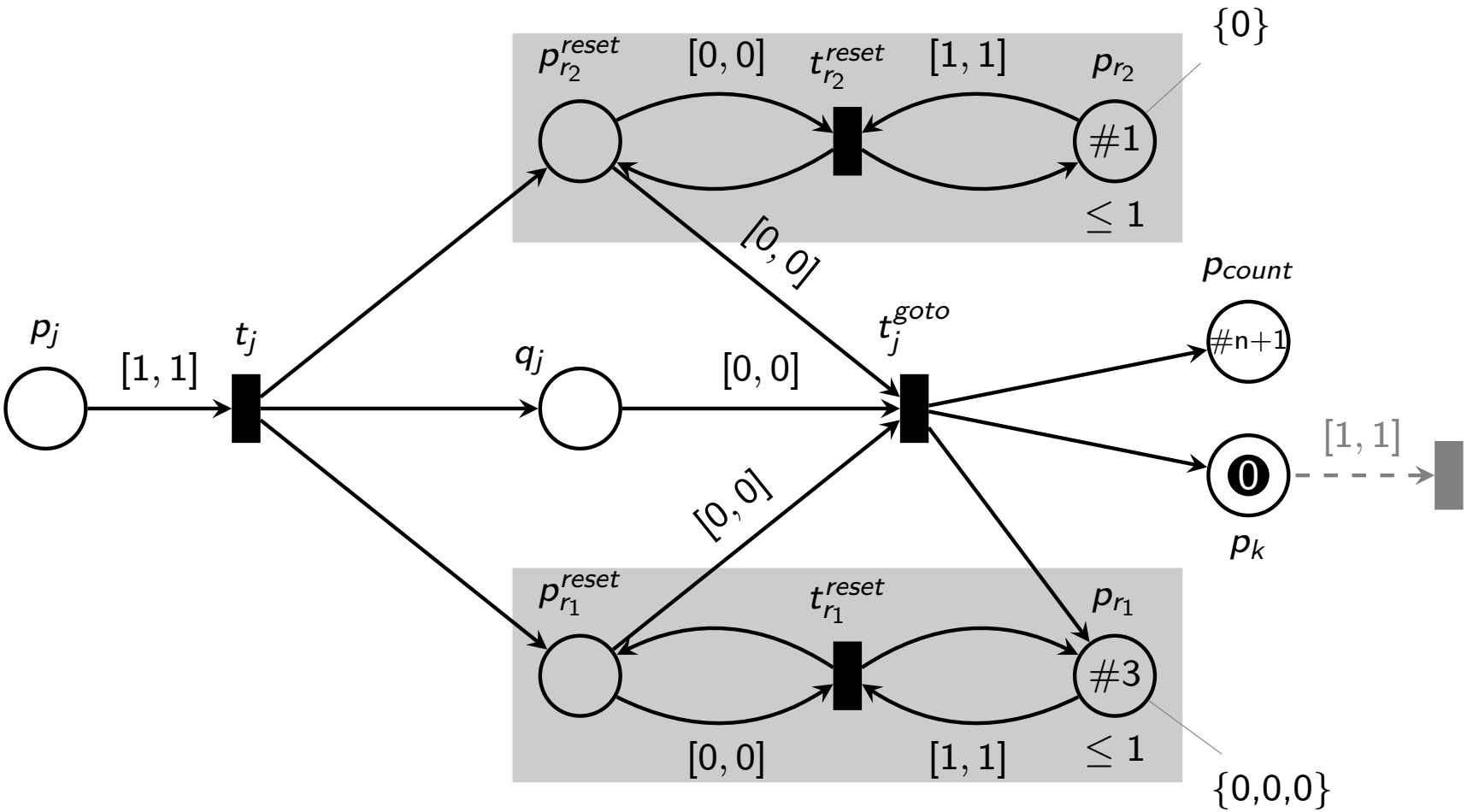

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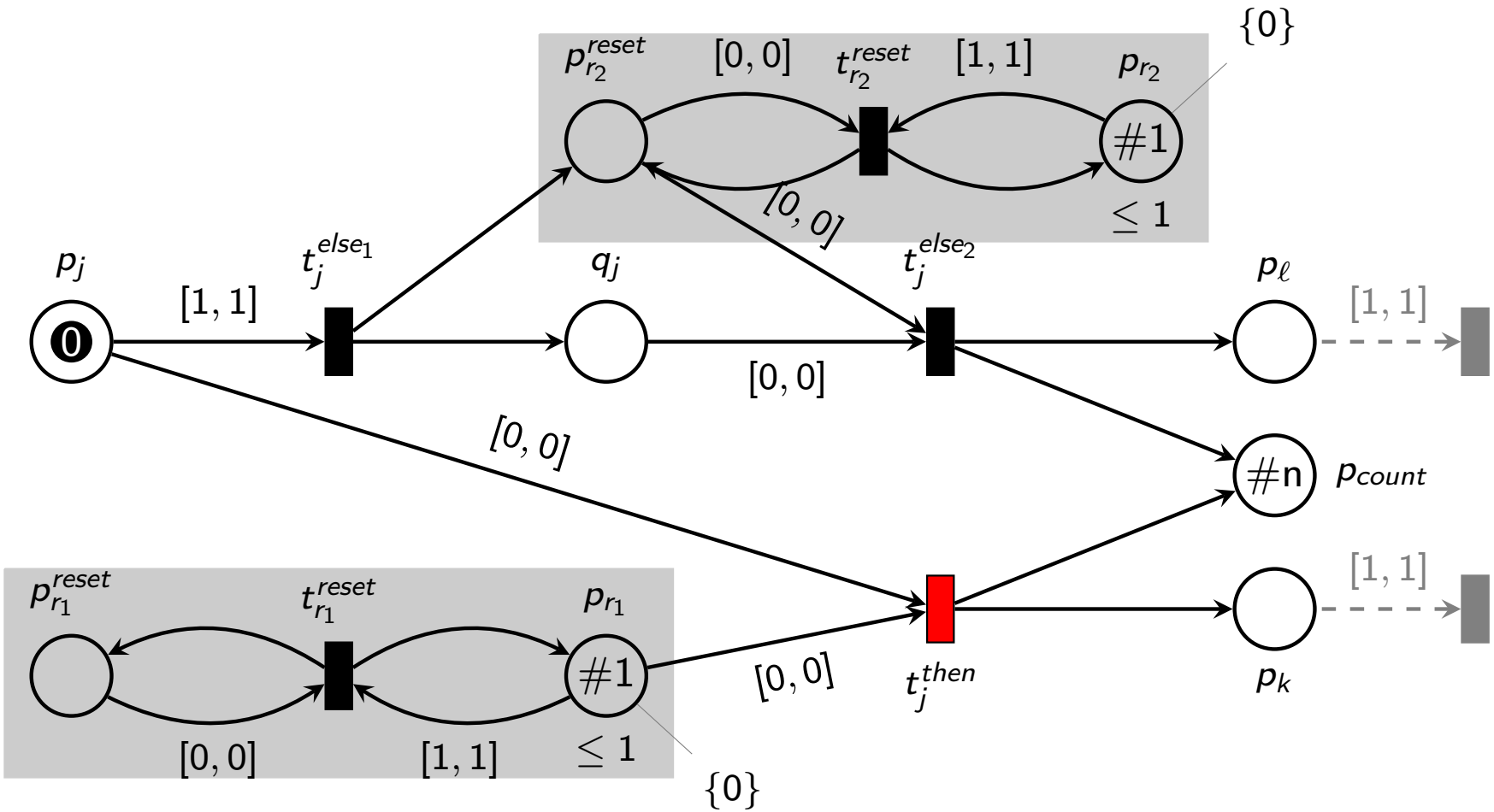

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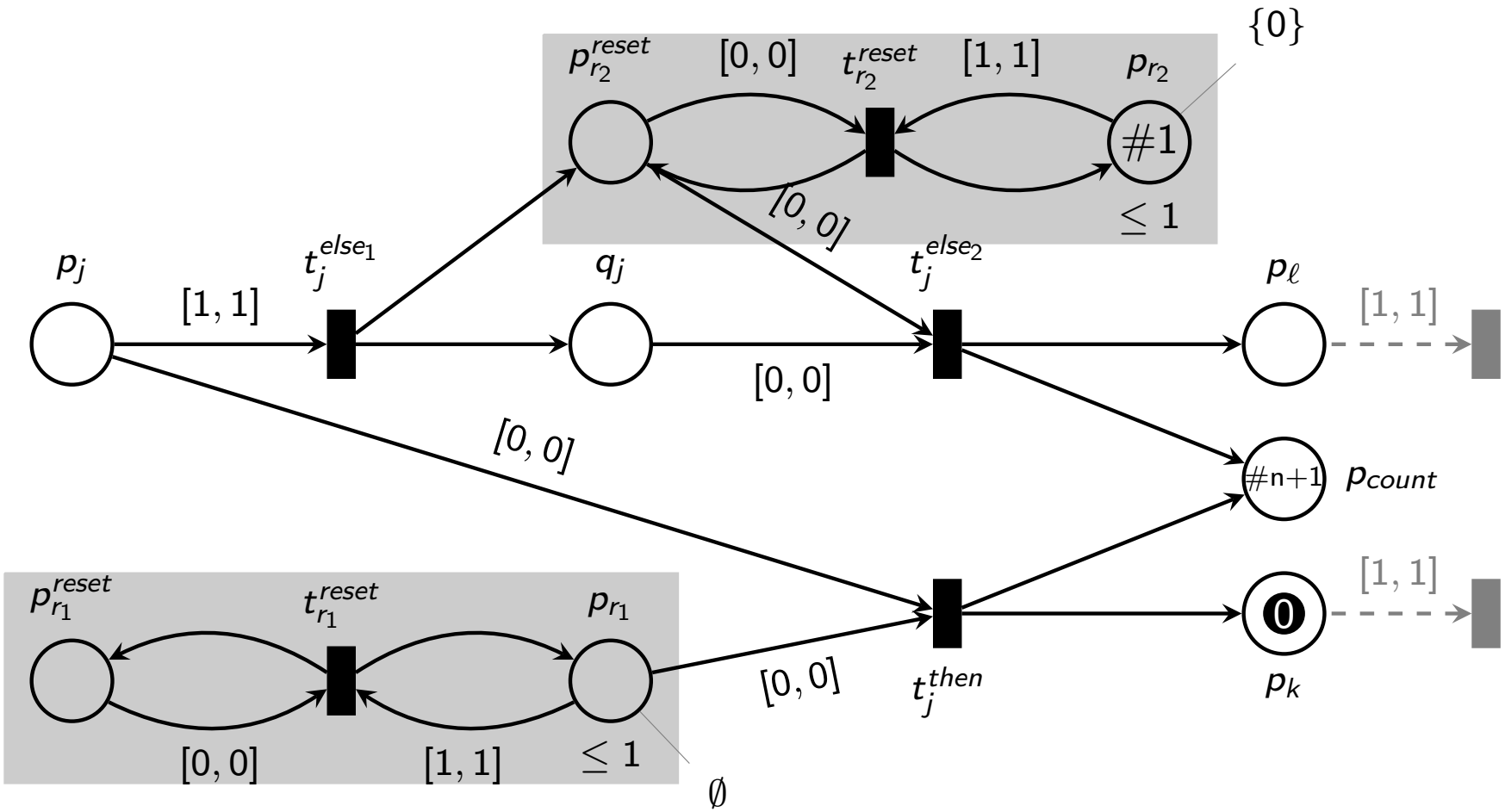

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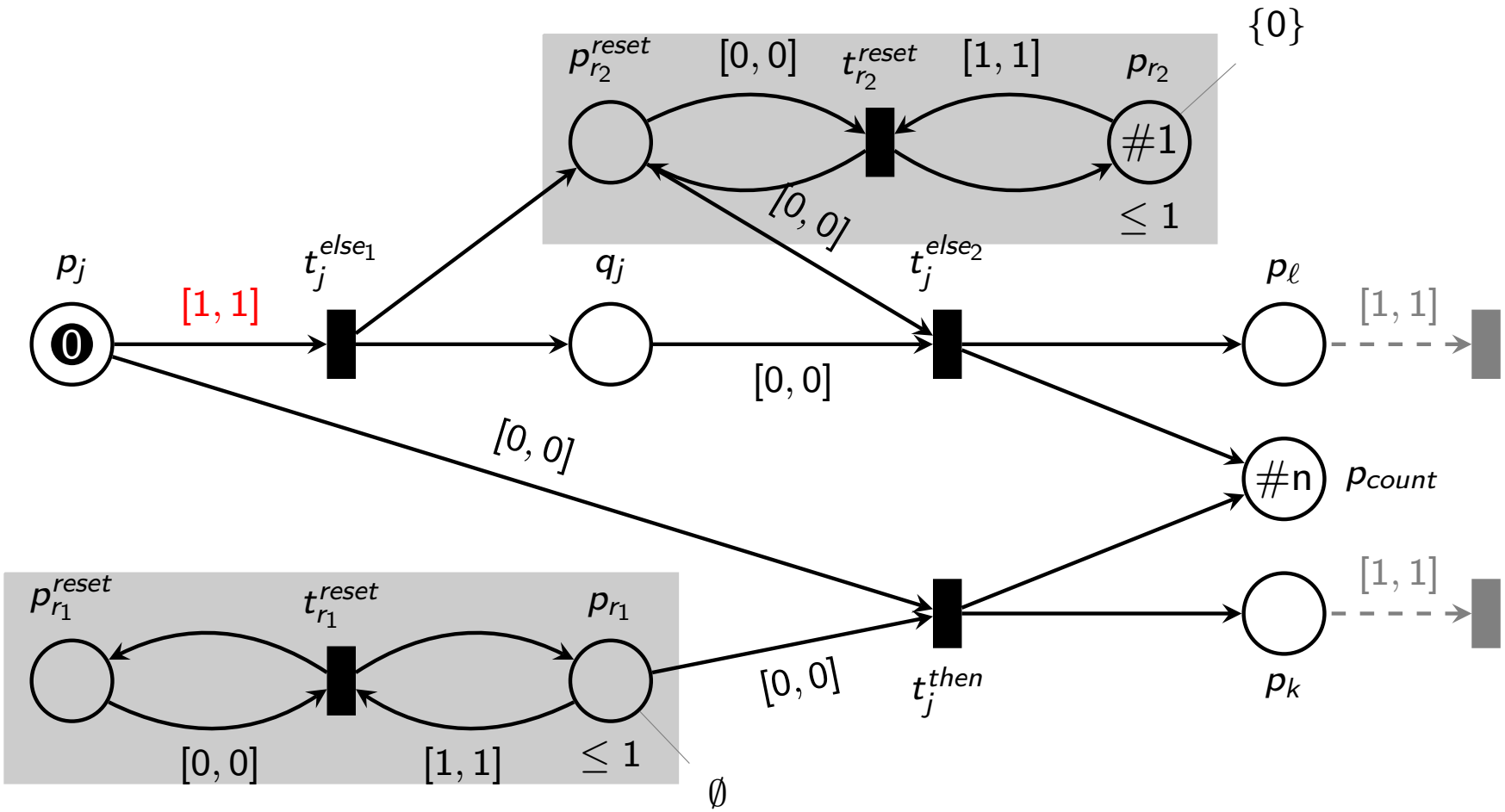

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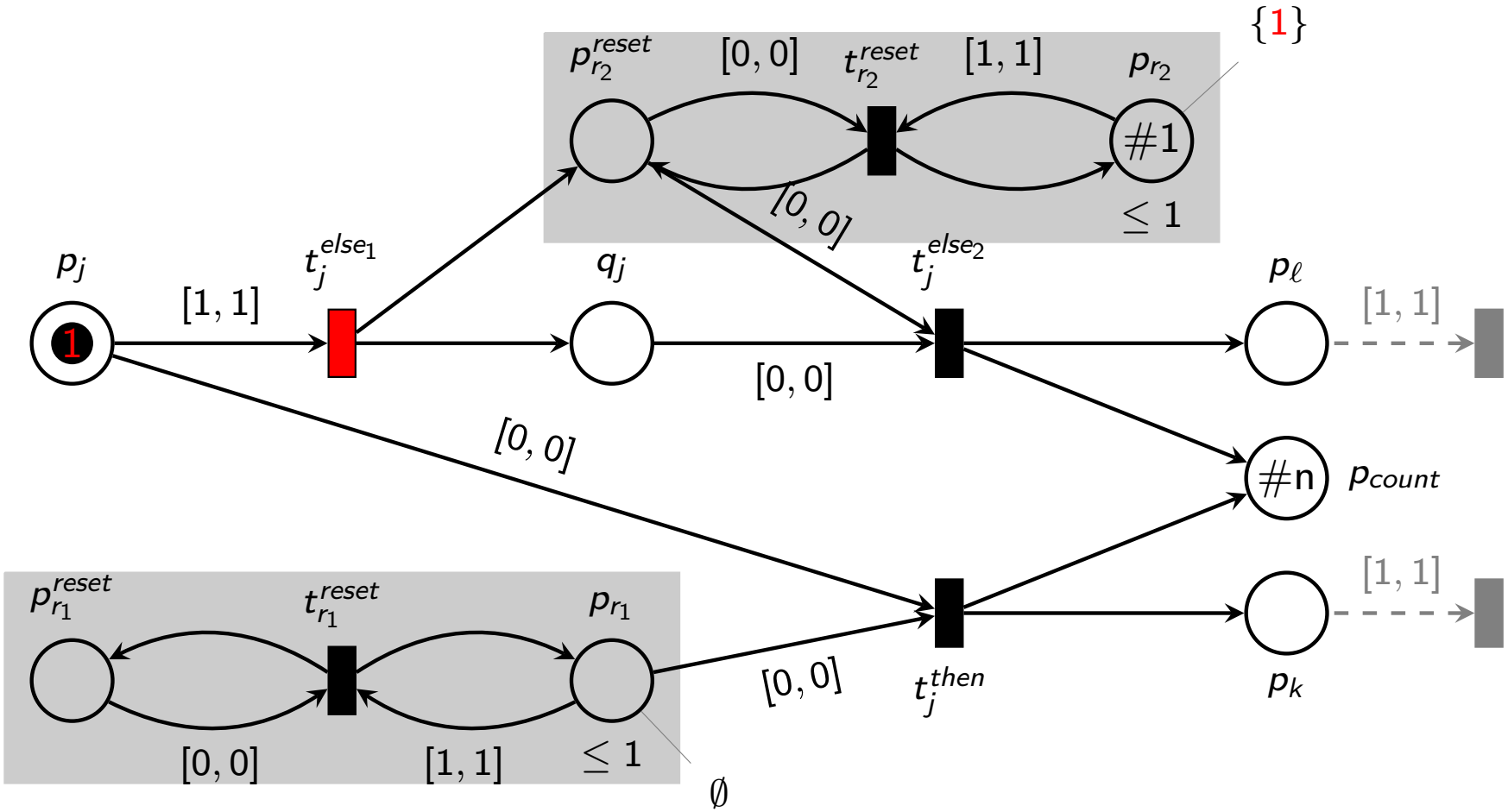

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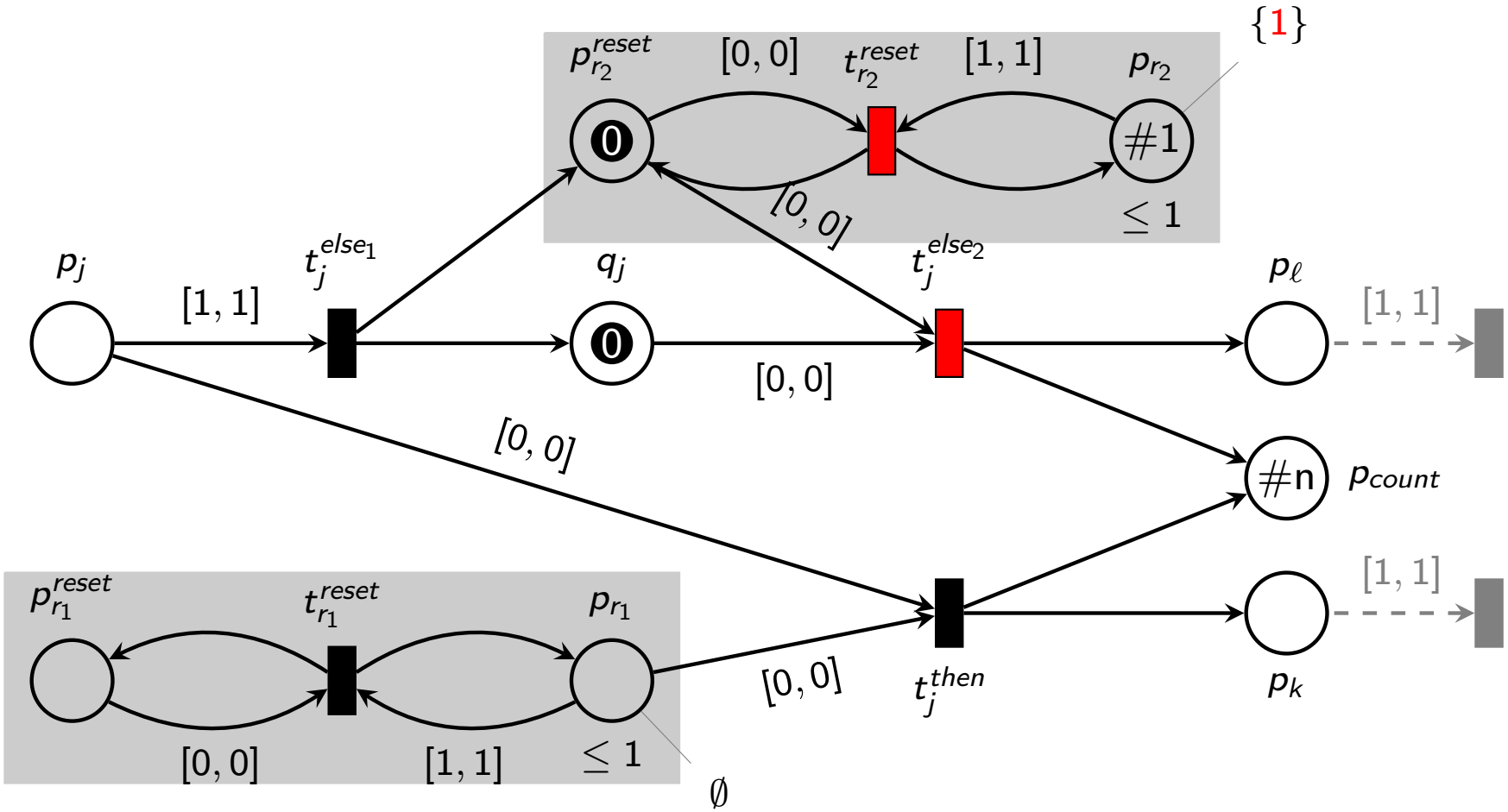

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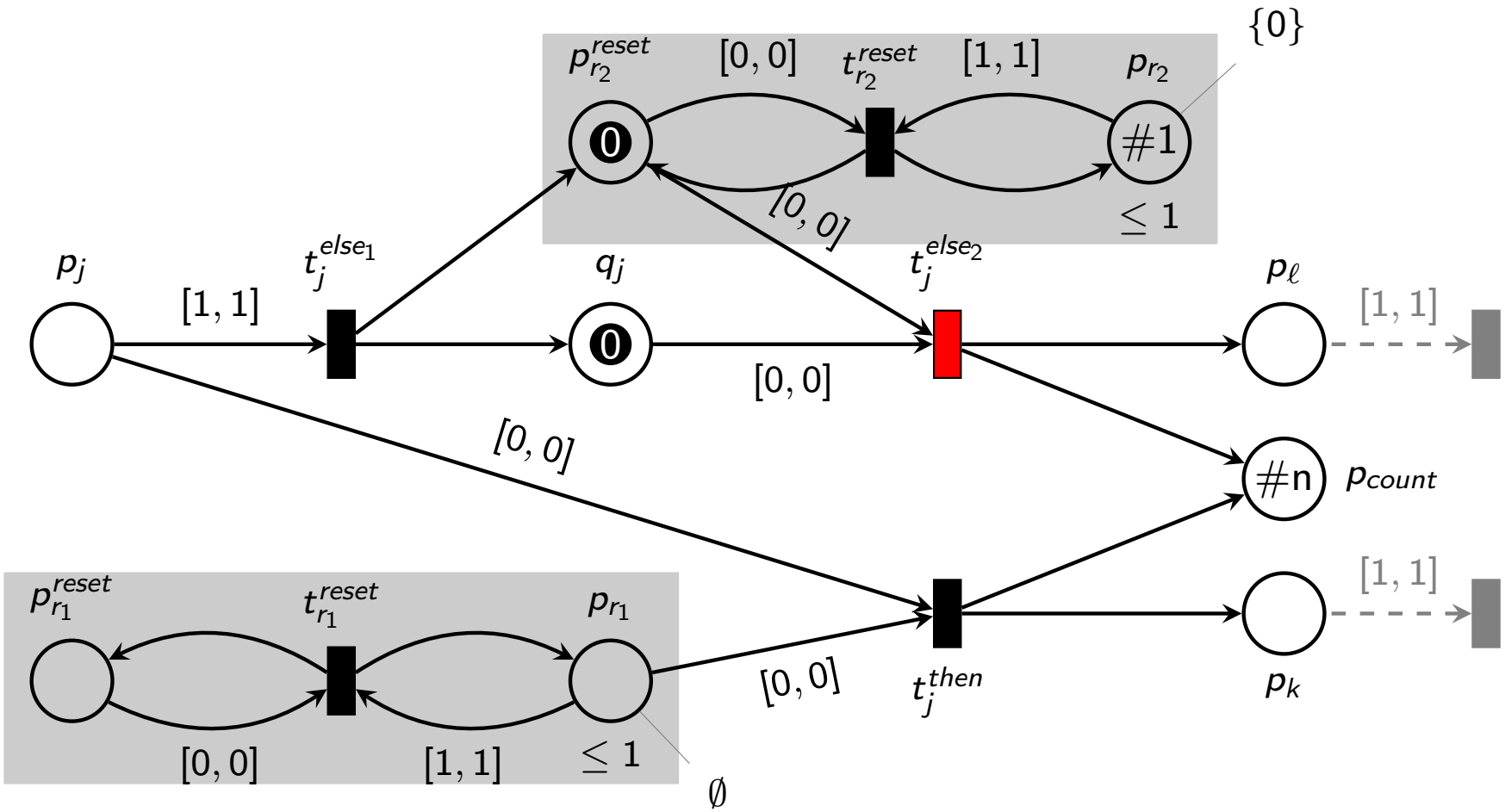

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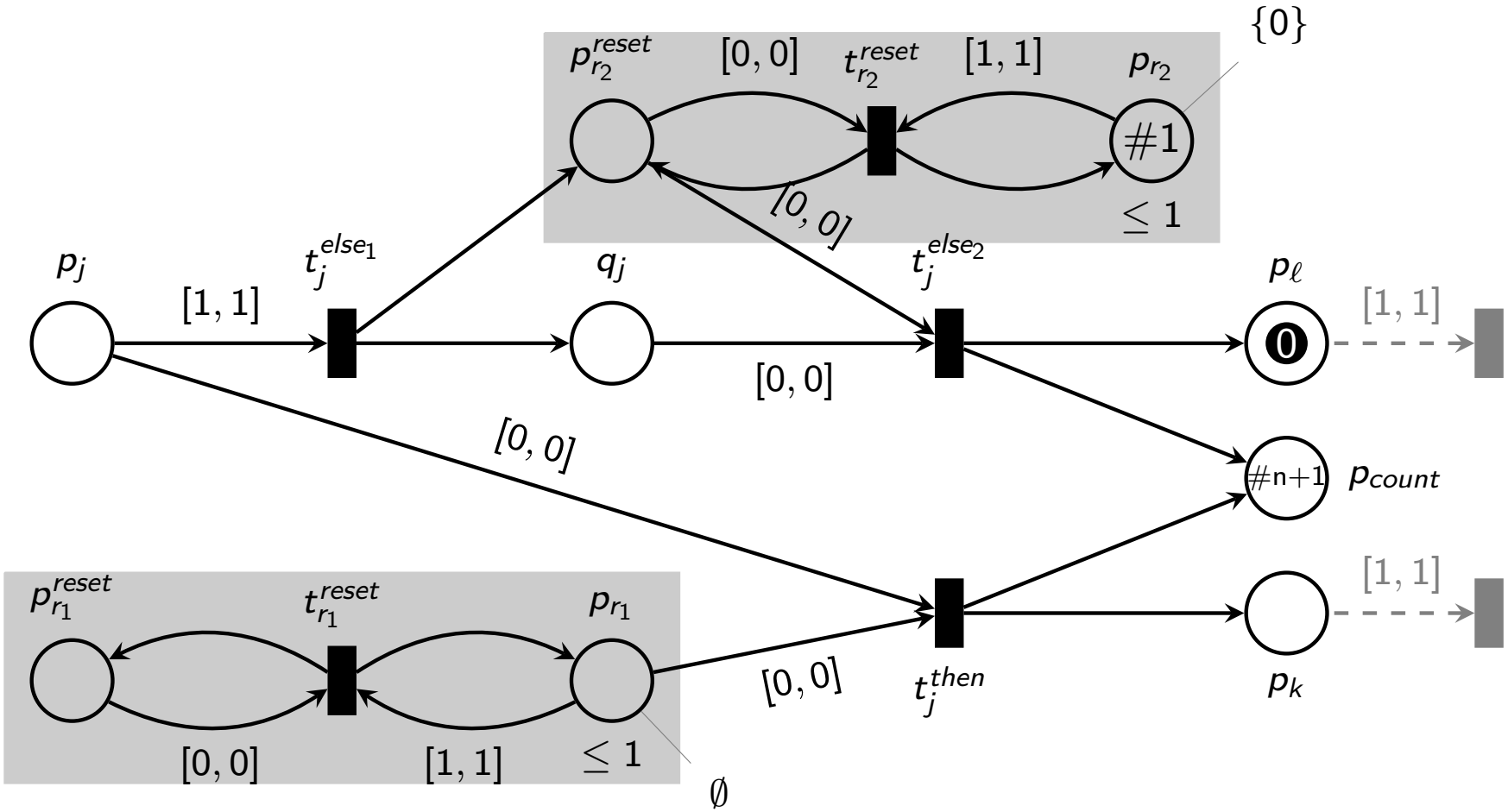

# Undecidability of Boundedness

**Lemma**

Given a 2-CM $CM$ and the associated ITAPN $(N, M_0)$,

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**Theorem**

The *Boundedness* problem is undecidable for Timed-Arc Petri Nets with Invariants.

# Undecidability of Coverability

**Lemma**

Let $M$ be a marking such that there is a token at $p_{halt}$ and none in any other place. Given a 2-CM $CM$ and the associated marked ITAPN $(N, M_0)$,

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**Theorem**

The *Coverability* problem is undecidable for Timed-Arc Petri Nets with Invariants.

# Conclusion

We have shown that *Boundedness* and *coverability* is both *undecidable* in Timed-arc Petri Nets with Invariants.

| | Reachability | Boundedness | Coverability |
|---|---|---|---|
| PN | decidable | decidable | decidable |
| TAPN | undecidable | decidable | decidable |
| ITAPN | undecidable | *undecidable* | *undecidable* |

# Discussion

How does TAPAAL deal with this result.

- TAPAAL can verify $k - boundedness$ as it is decidable.
- TAPAAL supports optimization of the bound $k$ if the net is bounded.
- If unbounded, then TAPAAL makes an under approximation.

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