Ota Jirák and Dušan Kolář
FIT Brno Unversity of Technology

Scattered Context Grammar via Lazy Function Evaluation

Nov 15th 2009

MEMICS 2009, Znojmo

Scattered Context Grammar

- SCG $G_1 = (V,T,P_1,S)$
- V – total alphabet
- T – terminal, $T \subset V$
- P_1 – productions
  $(A_1,\dots,A_n) \longrightarrow (w_1,\dots,w_n)$, $A_1 \dots A_n \in V \setminus T$,
  $w_1,\dots,w_n \in V^*$
- S – starting nonterminal, $S \in V \setminus T$

Motivation

- We are interested in deterministic SCG compilers. Thus we have to use LL/LR SCG grammars.
- We want to work only with the pushdown top.

State Of The Art

- Deep pushdown (DPDA) with certain limitation and modification.
- Regulated pushdown automata.

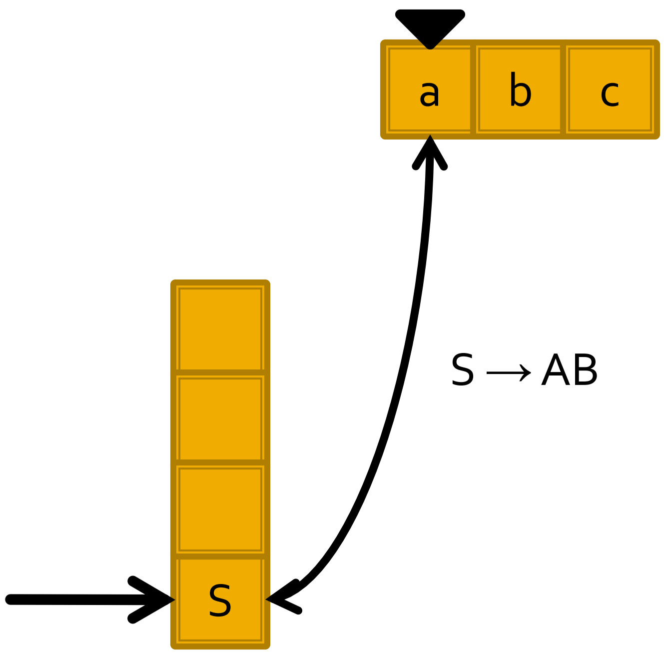
Basic Idea

- Process only necessary part of production and the rest mark to be processed later.

SCG

- SCG $G_1 = (V,T,P_1,S)$
- V – total alphabet
- T – terminal, $T \subset V$
- P_1 – productions
  $\mathbf{(A_1,\ldots,A_n) \longrightarrow (w_1,\ldots,w_n)}$,
  $A_1 \ldots A_n \in V \setminus T$,
  $w_1,\ldots,w_n \in V$
- S – starting nonterminal, $S \in V \setminus T$

CFG



CFG G1 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

p1: S → AB

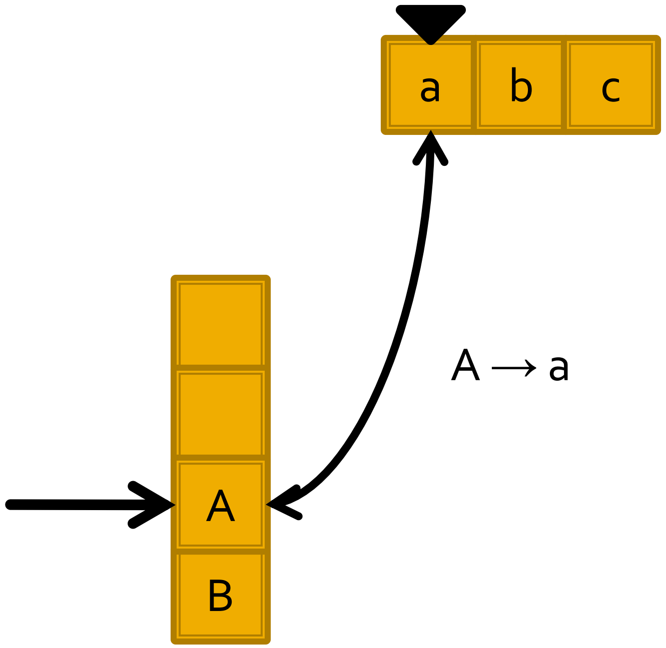
p2: A → a

p3: B → bC

p4: C → c

}

CFG



CFG G1 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

p1: S ⟶ AB

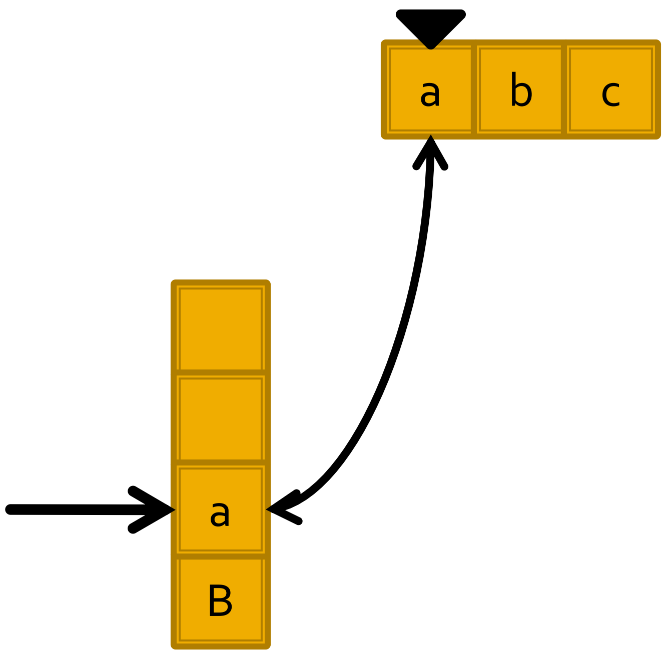
p2: A ⟶ a

p3: B ⟶ bC

p4: C ⟶ c

}

CFG



CFG G1 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

p1: $S \longrightarrow AB$

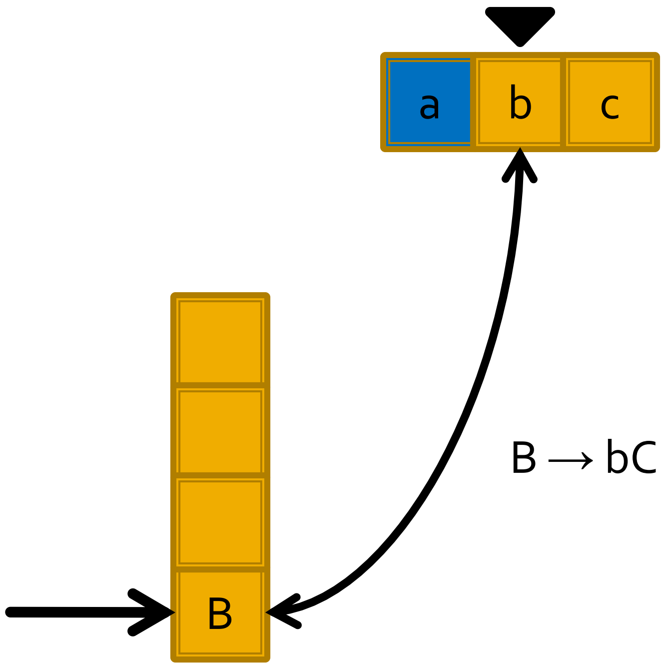
p2: $A \longrightarrow a$

p3: $B \longrightarrow bC$

p4: $C \longrightarrow c$

}

CFG



CFG G1 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

p1: S ⟶ AB

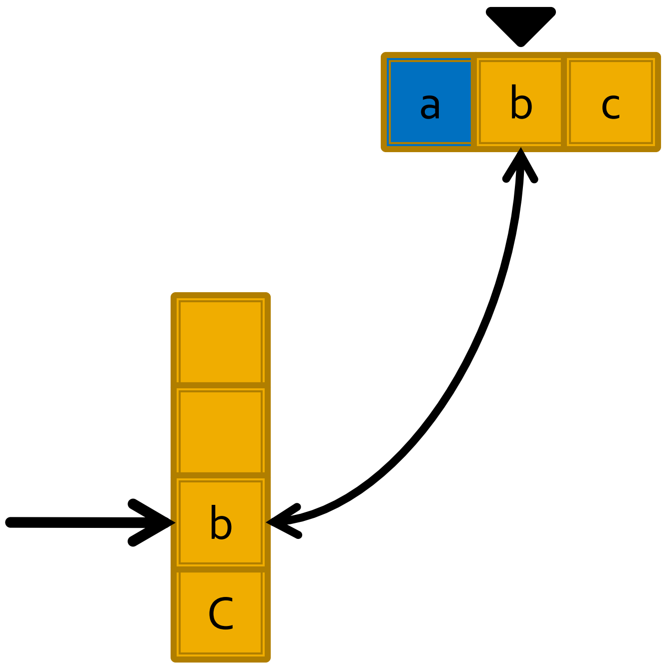
p2: A ⟶ a

p3: B ⟶ bC

p4: C ⟶ c

}

CFG



CFG G1 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

p1: $S \longrightarrow AB$

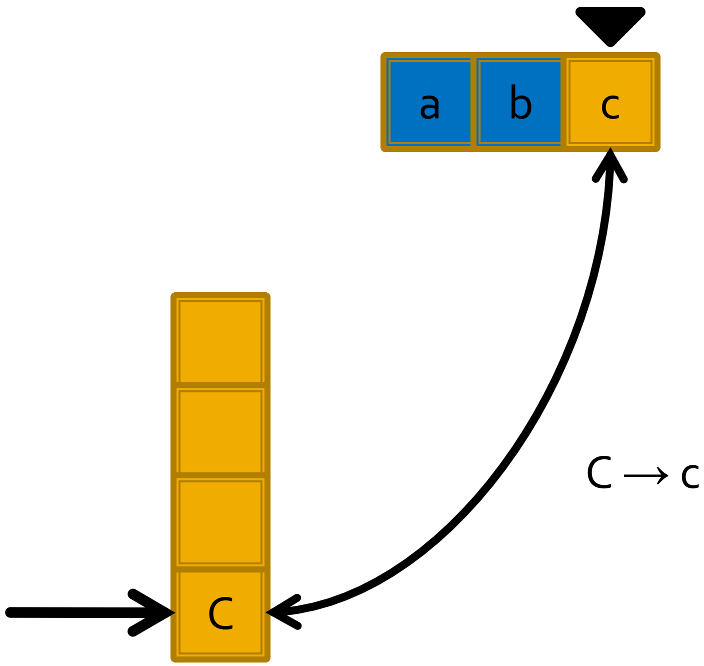
p2: $A \longrightarrow a$

p3: $B \longrightarrow bC$

p4: $C \longrightarrow c$

}

CFG



CFG G1 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

p1: $S \longrightarrow AB$

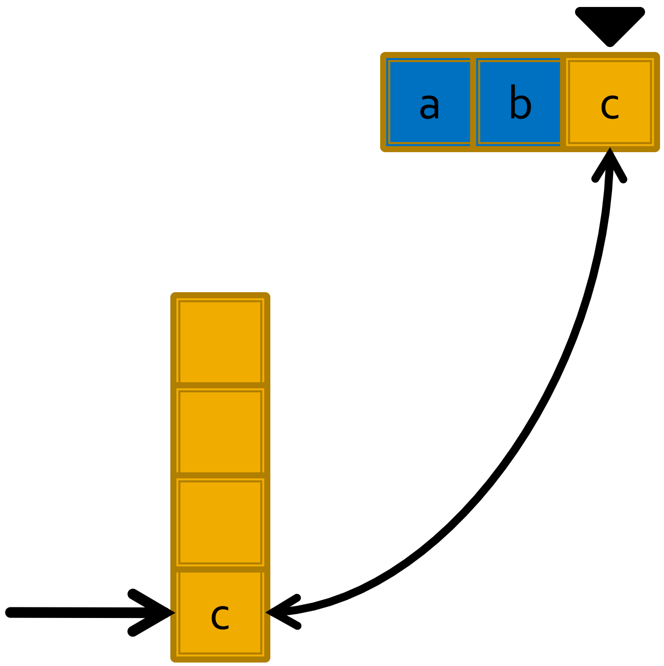
p2: $A \longrightarrow a$

p3: $B \longrightarrow bC$

p4: $C \longrightarrow c$

}

CFG



CFG G1 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

p1: S ⟶ AB

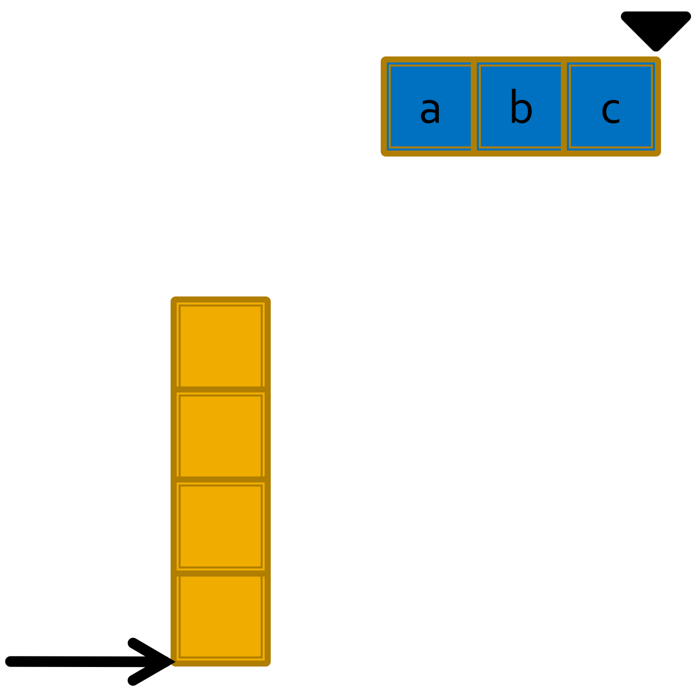
p2: A ⟶ a

p3: B ⟶ bC

p4: C ⟶ c

}

CFG



CFG G1 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

p1: $S \longrightarrow AB$

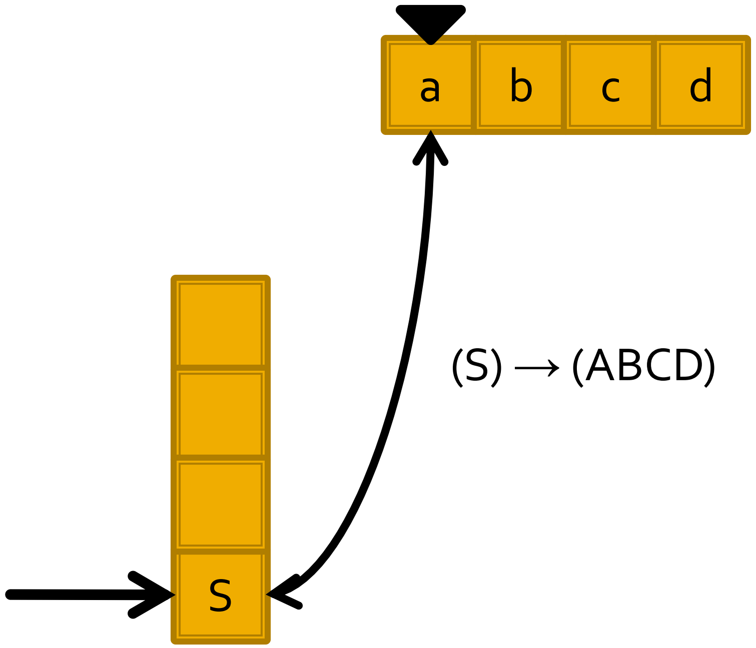
p2: $A \longrightarrow a$

p3: $B \longrightarrow bC$

p4: $C \longrightarrow c$

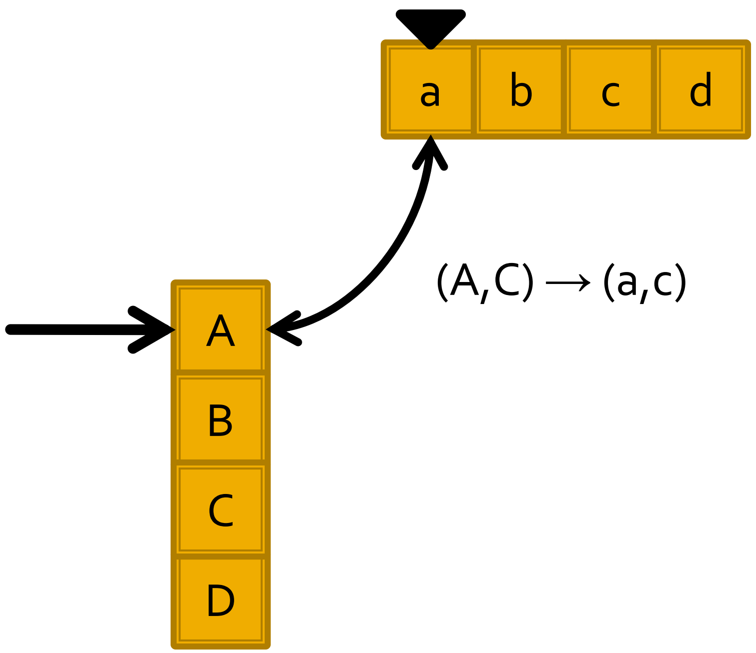
}

SCG



SCG G2 = (V,T,P,S)
V = {a,b,c,A,B,C,S}
T = {a,b,c}
P =
{
    p1: (S) ⟶ (ABCD)
    p2: (A,C) ⟶ (a,c)
    p3: (B,D) ⟶ (b,d)
}

SCG



SCG G2 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

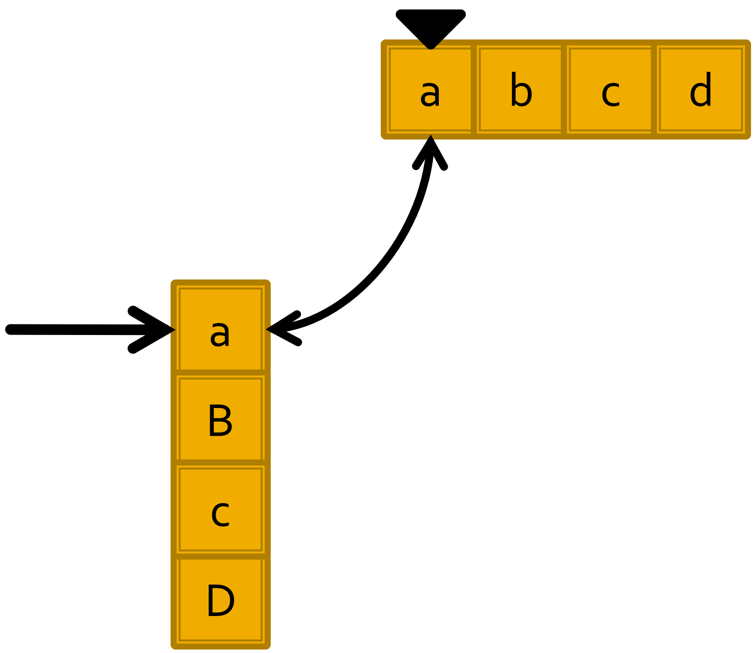
p1: (S) ⟶ (ABCD)

p2: (A,C) ⟶ (a,c)

p3: (B,D) ⟶ (b,d)

}

SCG



SCG G2 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

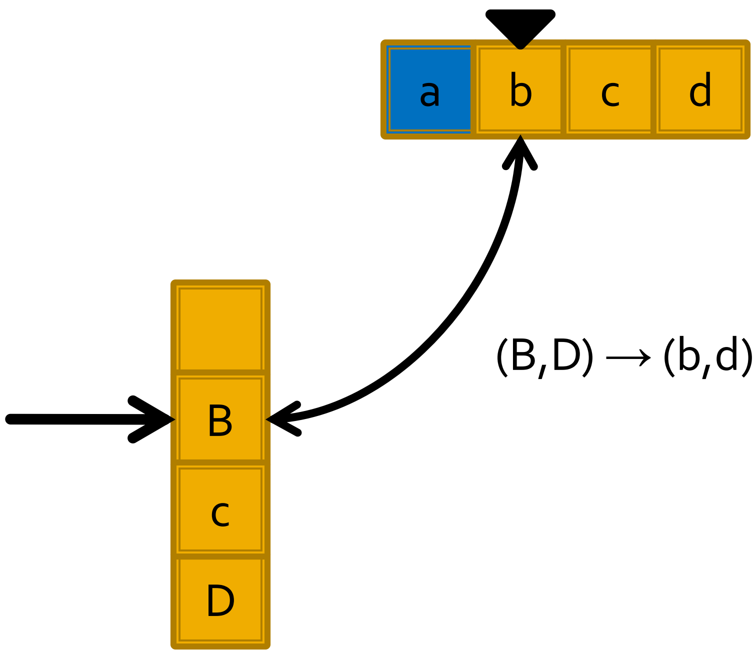
p1: (S) ⟶ (ABCD)

p2: (A,C) ⟶ (a,c)

p3: (B,D) ⟶ (b,d)

}

SCG



SCG G2 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

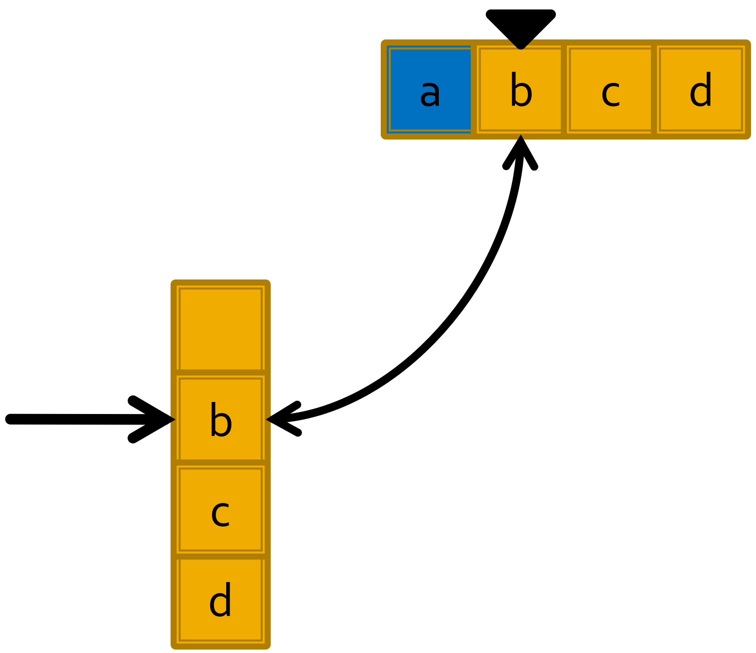
p1: (S) ⟶ (ABCD)

p2: (A,C) ⟶ (a,c)

p3: (B,D) ⟶ (b,d)

}

SCG



SCG G2 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

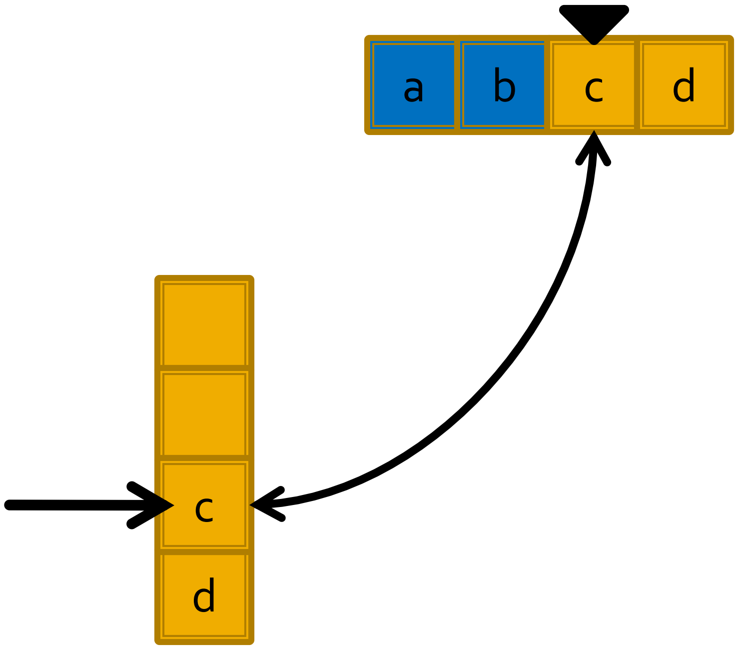
p1: (S) ⟶ (ABCD)

p2: (A,C) ⟶ (a,c)

p3: (B,D) ⟶ (b,d)

}

SCG



SCG G2 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

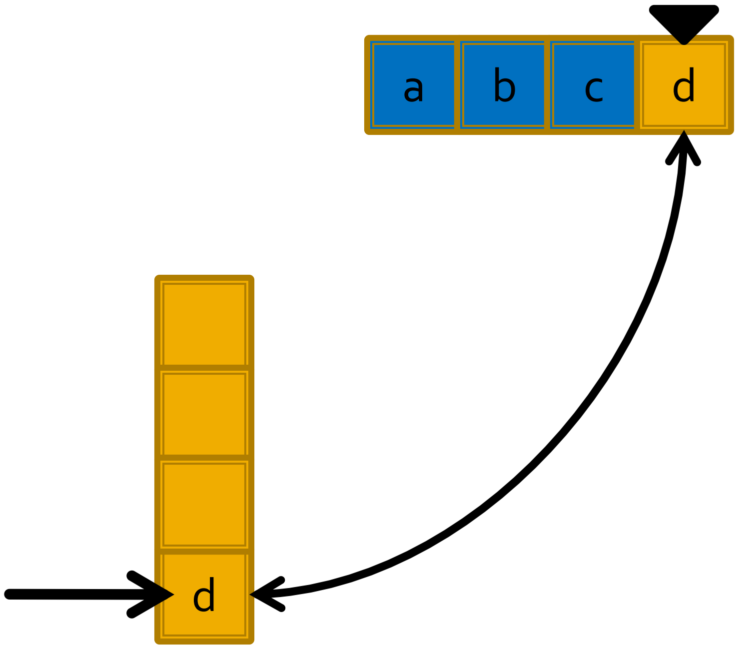
p1: (S) ⟶ (ABCD)

p2: (A,C) ⟶ (a,c)

p3: (B,D) ⟶ (b,d)

}

SCG



SCG G2 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

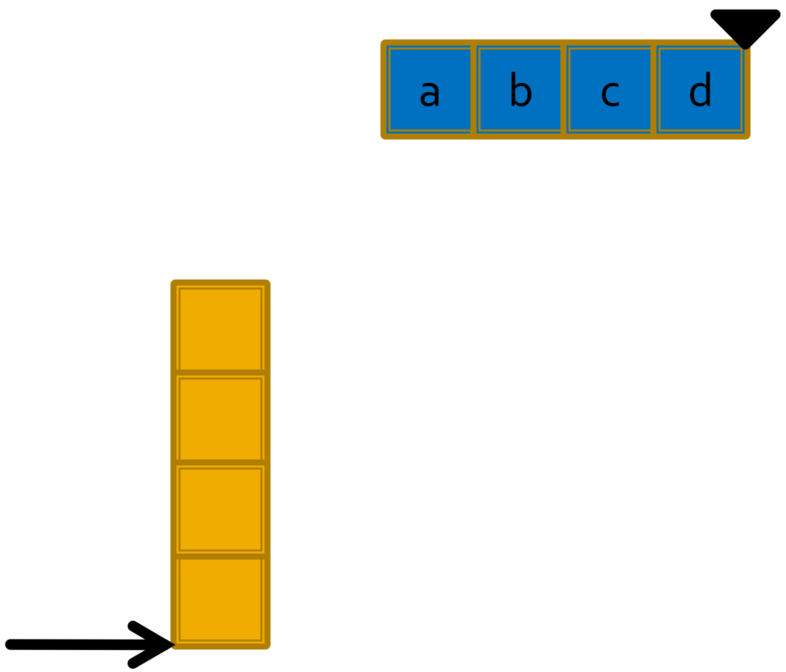
p1: (S) ⟶ (ABCD)

p2: (A,C) ⟶ (a,c)

p3: (B,D) ⟶ (b,d)

}

SCG



SCG G2 = (V,T,P,S)

V = {a,b,c,A,B,C,S}

T = {a,b,c}

P =

{

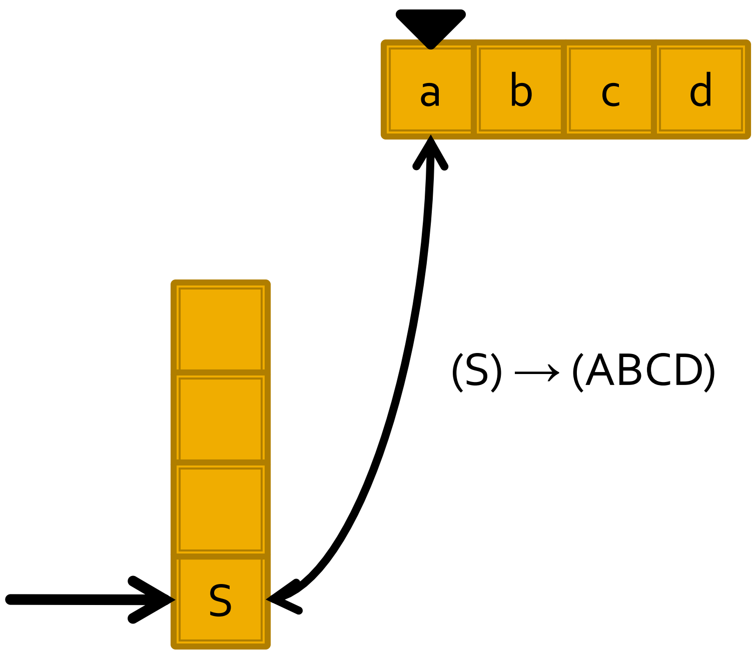
p1: (S) ⟶ (ABCD)

p2: (A,C) ⟶ (a,c)

p3: (B,D) ⟶ (b,d)

}

Example



Productions :

p1: (S) → (ABCD)

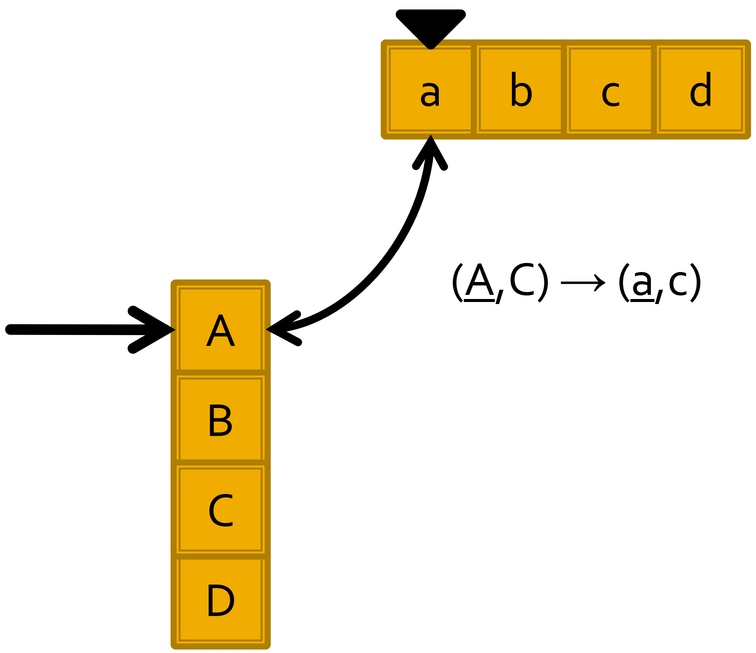
p2: (A,C) → (a,c)

p3: (B,D) → (b,d)

Delayed productions:

Example

p1

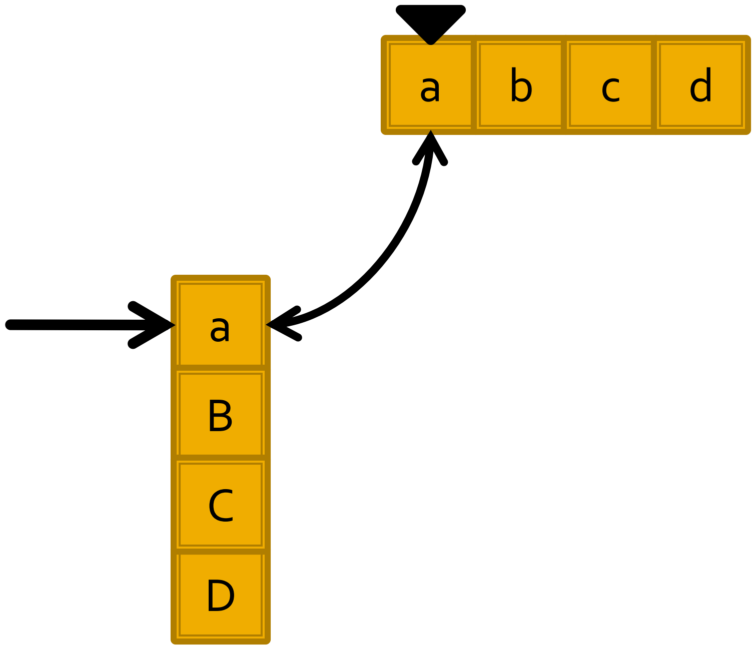


Productions:
p1: (S) → (ABCD)
p2: (A,C) → (a,c)
p3: (B,D) → (b,d)

Delayed productions:

Example

p1



Productions :

p1: (S) ⟶ (ABCD)

p2: (A,C) ⟶ (a,c)

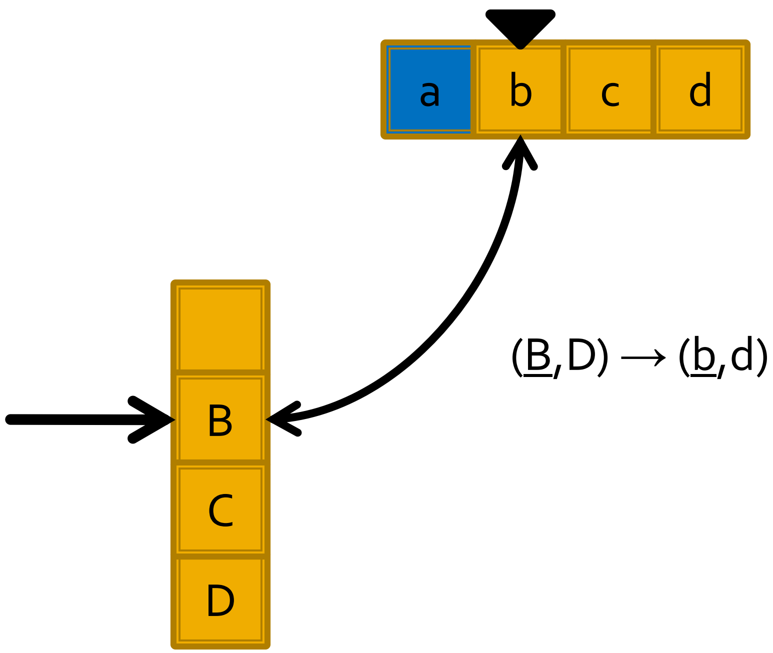
p3: (B,D) ⟶ (b,d)

Delayed productions:

(C) ⟶ (c) part of p2

Example

p1 p2

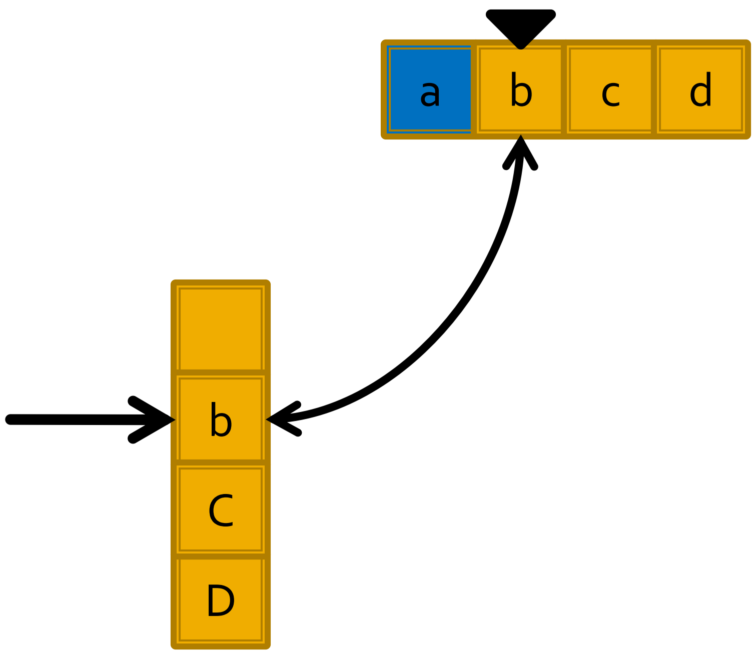


Productions :
p1: (S) ⟶ (ABCD)
p2: (A,C) ⟶ (a,c)
p3: (B,D) ⟶ (b,d)

Delayed productions:
(C) ⟶ (c) part of p2

Example

p1 p2



Productions :

p1: (S) ⟶ (ABCD)

p2: (A,C) ⟶ (a,c)

p3: (B,D) ⟶ (b,d)

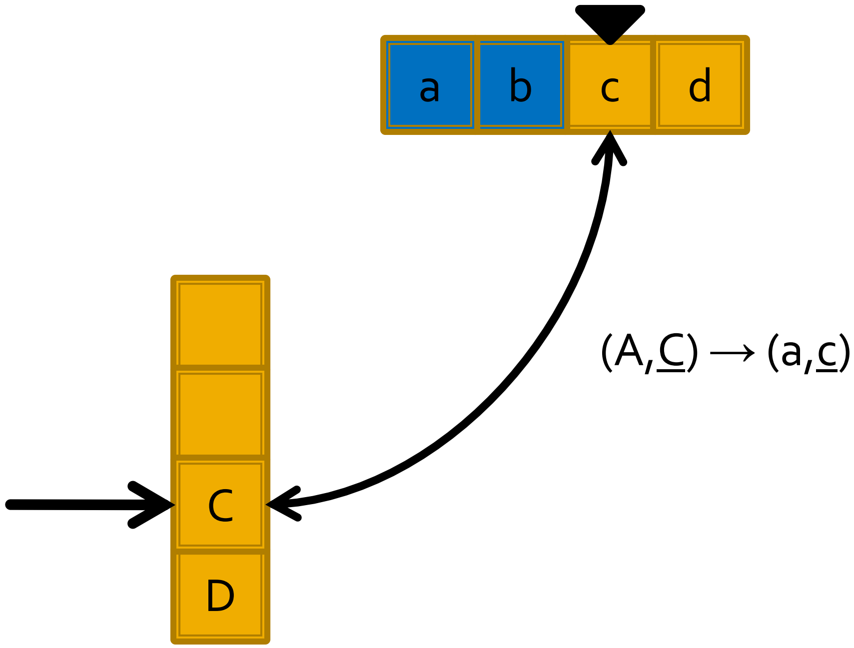
Delayed productions:

(C) ⟶ (c) part of p2

(D) ⟶ (d) part of p3

Example

p1 p2 p3

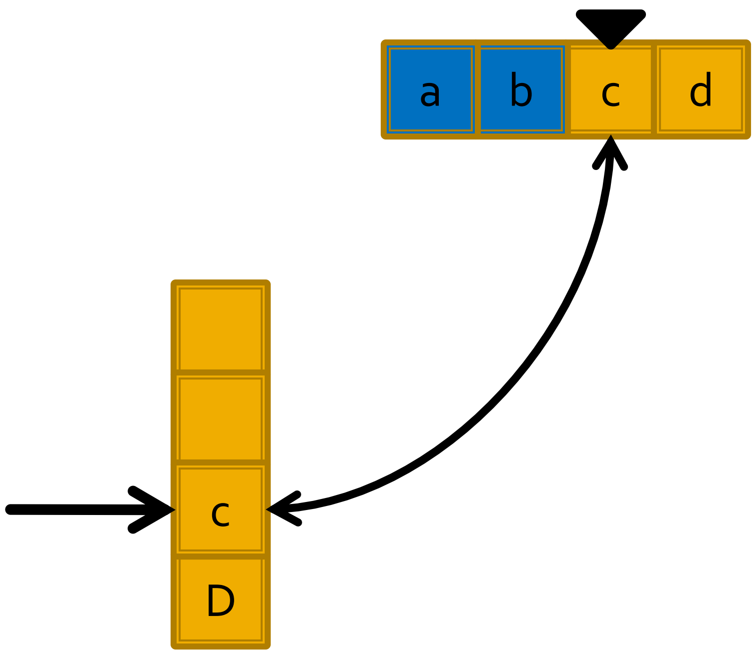


Productions :
p1: (S) → (ABCD)
p2: (A,C) → (a,c)
p3: (B,D) → (b,d)

Delayed productions:
(C) → (c) part of p2
(D) → (d) part of p3

Example

p1 p2 p3

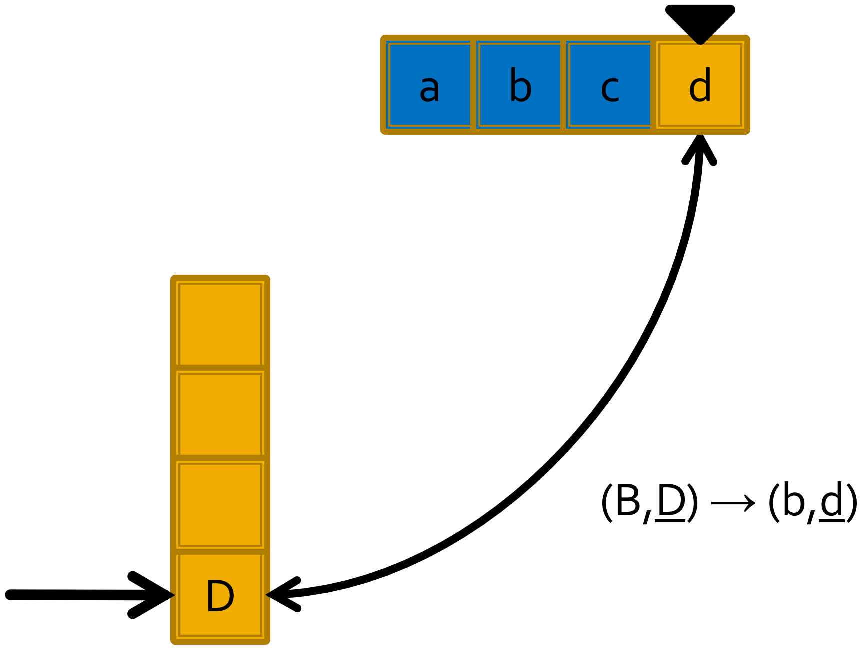


Productions :
p1: (S) ⟶ (ABCD)
p2: (A,C) ⟶ (a,c)
p3: (B,D) ⟶ (b,d)

Delayed productions:
(D) ⟶ (d) part of p3

Example

p1 p2 p3

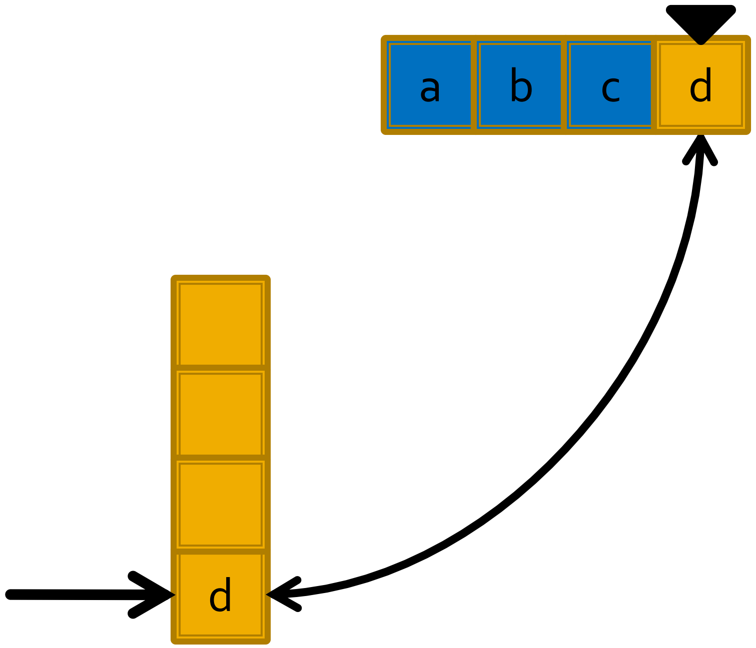


Productions :
p1: (S) ⟶ (ABCD)
p2: (A,C) ⟶ (a,c)
p3: (B,D) ⟶ (b,d)

Delayed productions:
(D) ⟶ (d) part of p3

Example

p1 p2 p3



Productions :

p1: (S) ⟶ (ABCD)

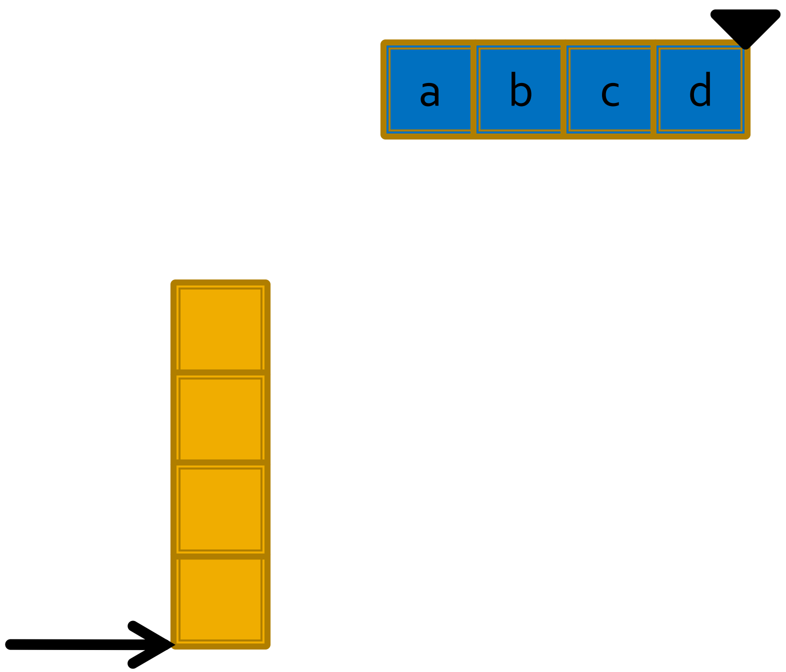
p2: (A,C) ⟶ (a,c)

p3: (B,D) ⟶ (b,d)

Delayed productions:

Example

p1 p2 p3



Productions :

p1: (S) ⟶ (ABCD)

p2: (A,C) ⟶ (a,c)

p3: (B,D) ⟶ (b,d)

Delayed productions:

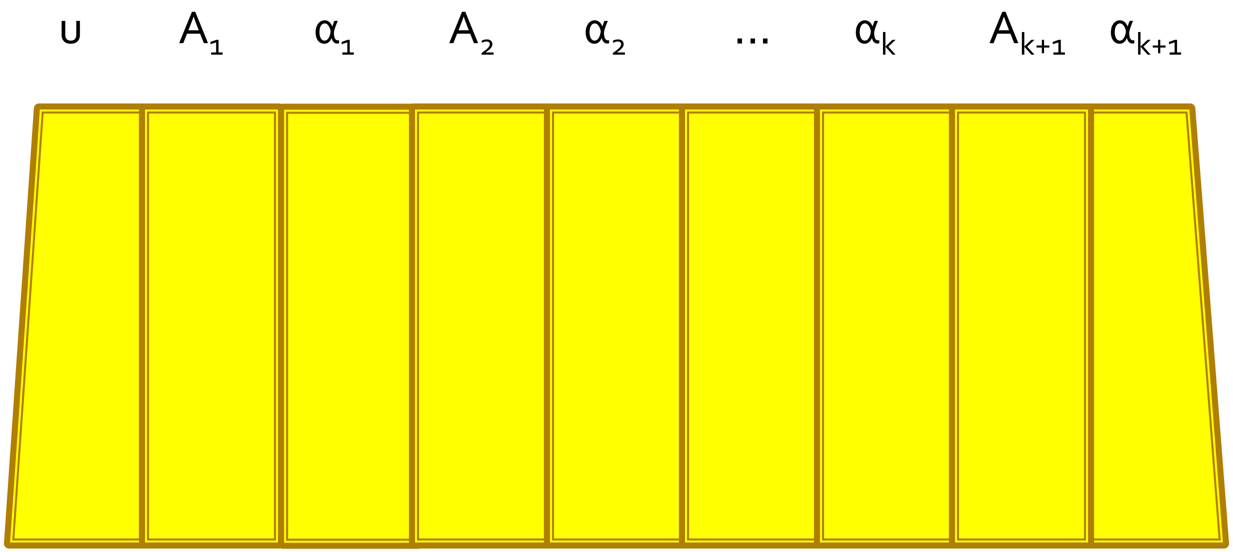
Regular Derivation

- Let $X = x_1A_1x_2A_2 \ldots x_nA_nx_{n+1}$ be a sentential form, $A_i \in V \setminus T$, $x_i \in (V \setminus \{A_i\})^*$, $x_{n+1} \in V^*$, for some $n \geq 1$, j is a number of derivation step and $p_j : (A_1, A_2, \ldots, A_n) \longrightarrow (w_1, w_2, \ldots, w_n) \in P$ is an SCG production used in the j-th derivation step.

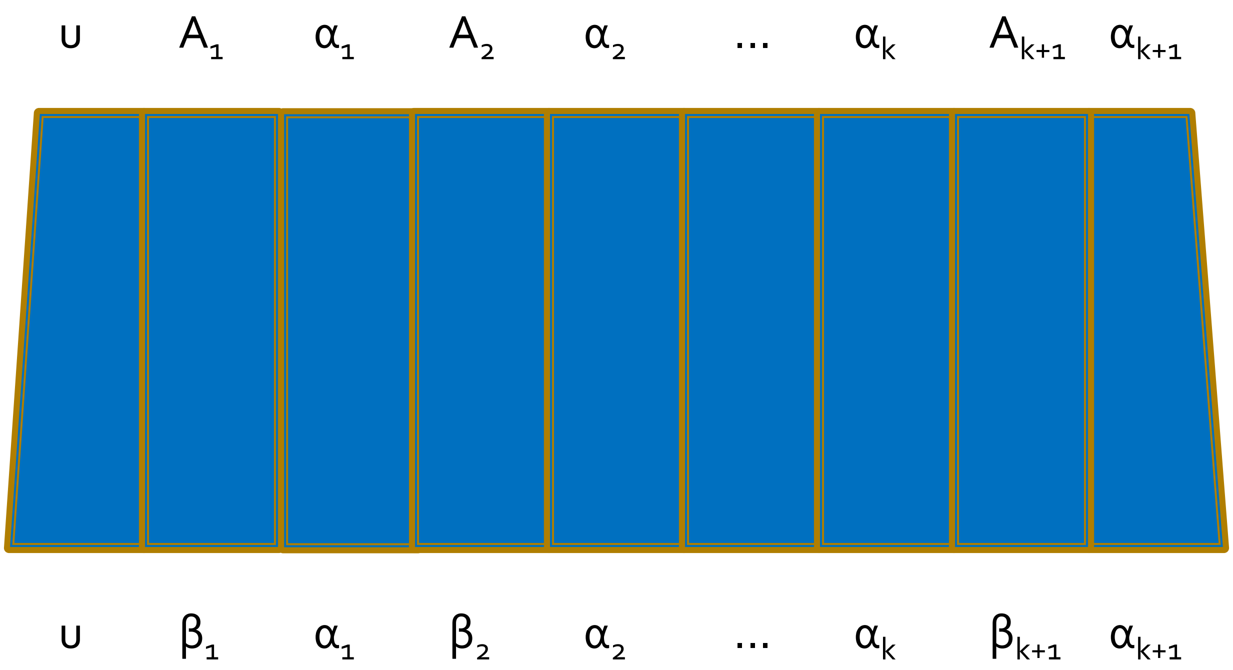
  Function $h_j(X)$ stands for the leftmost application of the SCG production used in the j-th derivation step that is:

- $h_j(X) = h_j(x_1A_1x_2A_2 \ldots x_nA_nx_{n+1}) = x_1w_1x_2w_2 \ldots x_nw_nx_{n+1}$

Regular Derivation - Example



Regular Derivation - Example



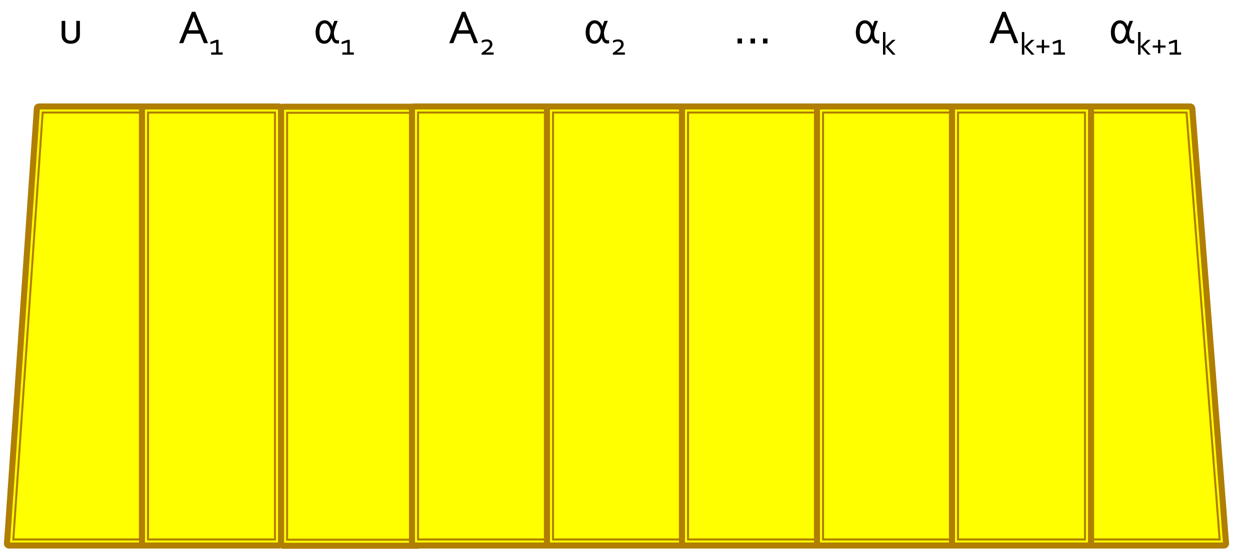
Delayed Derivation Step 1/2

- Let $X = x_1A_1x_2A_2 \ldots x_nA_nx_{n+1}$ be a sentential form, $A_i \in V \setminus T$, $x_i \in (V \setminus \{A_i\})^*$, $x_{n+1} \in V^*$, for some $n \geq 1$, j is a number of derivation step and $p_j : (A_1, A_2, \ldots, A_n) \longrightarrow (w_1, w_2, \ldots, w_n) \in P$ is an SCG production used in the j-th derivation step, $m \in \{1, \ldots, n + 1\}$.

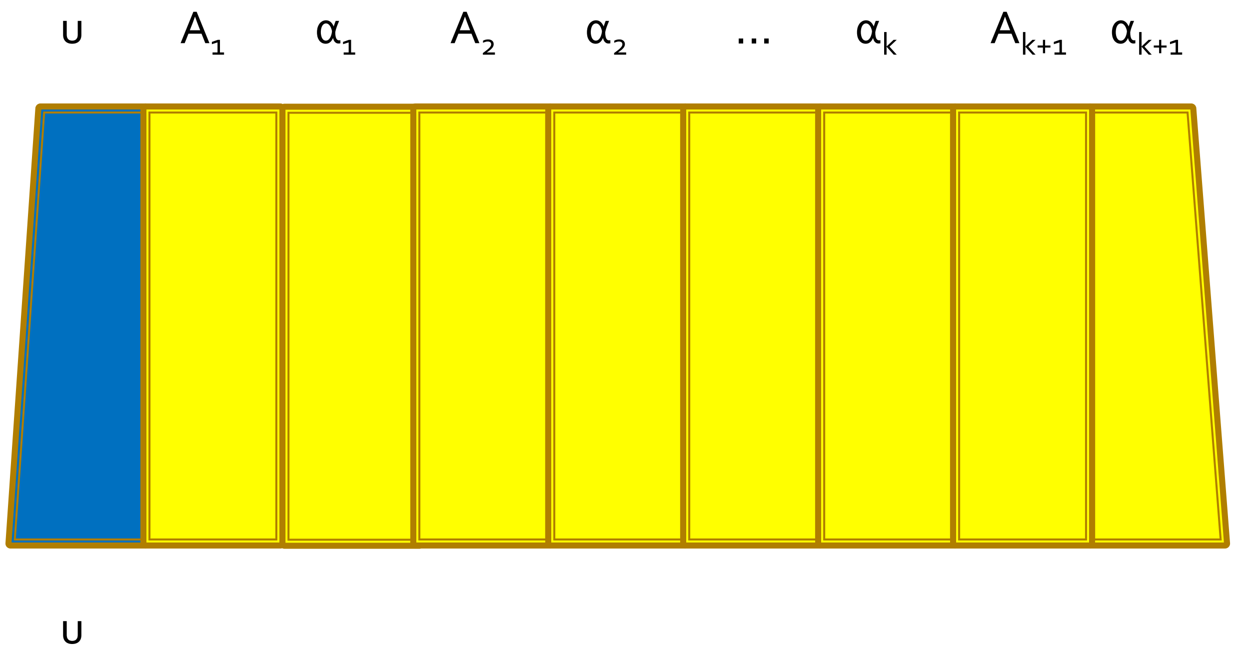
Delayed Derivation Step 2/2

- $g_j(m,X) =$
  - $w_m g_j(m + 1, X[2:])$ for $m \leq n, X[1] = A_m$,
  - $X[1] g_j(m, X[2:])$ for $m \leq n, X[1] \,!= A_m$,
  - ε for $m > n, |X| = 0$,
  - $X[1]\, g_j(m, X[2:])$ for $m > n, |X| > 1$,
  - $X[1]$ for $m > n, |X| = 1$.

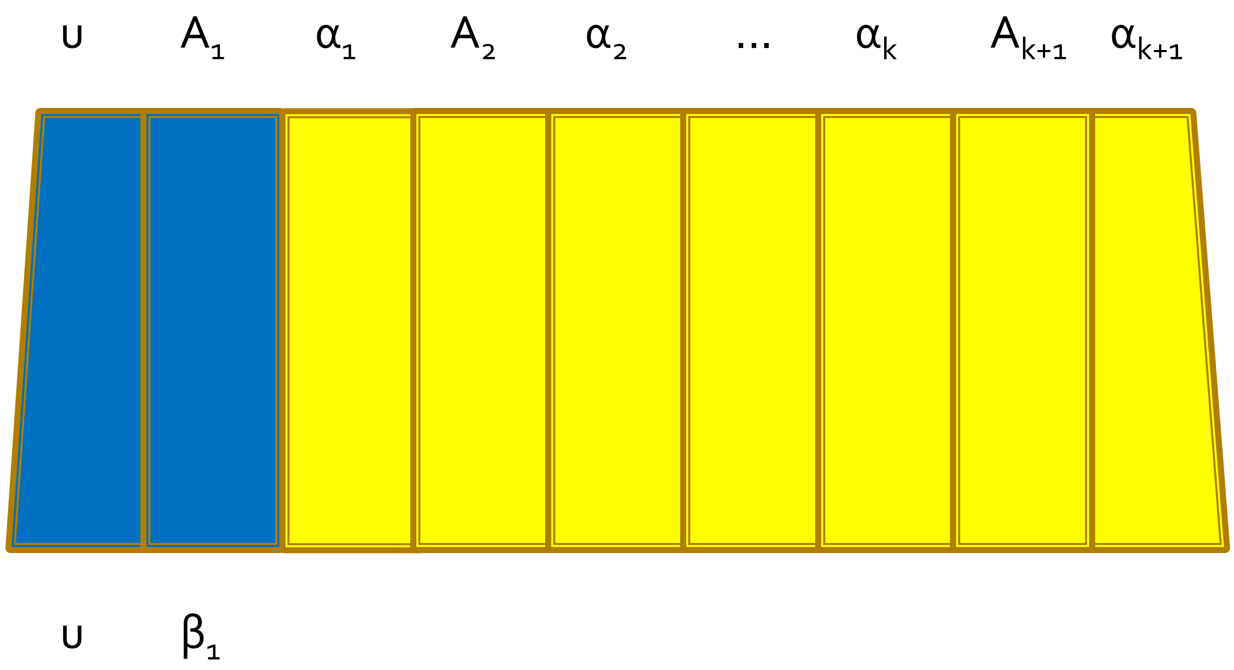
Delayed Derivation - Example



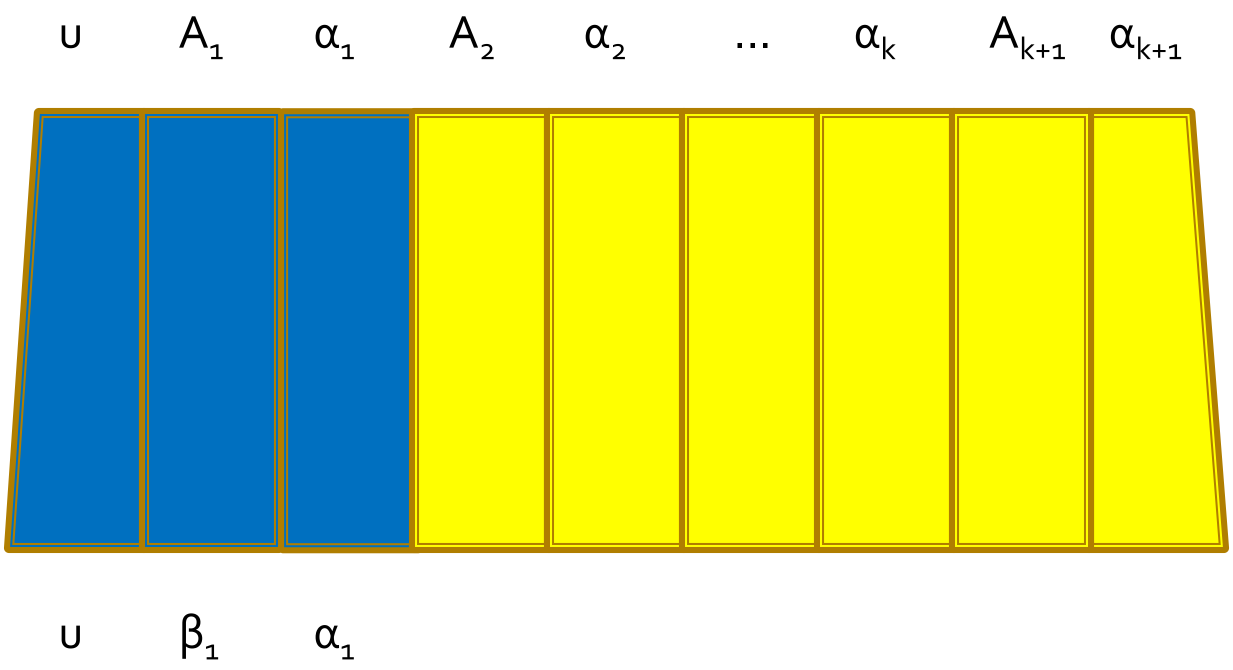
Delayed Derivation - Example



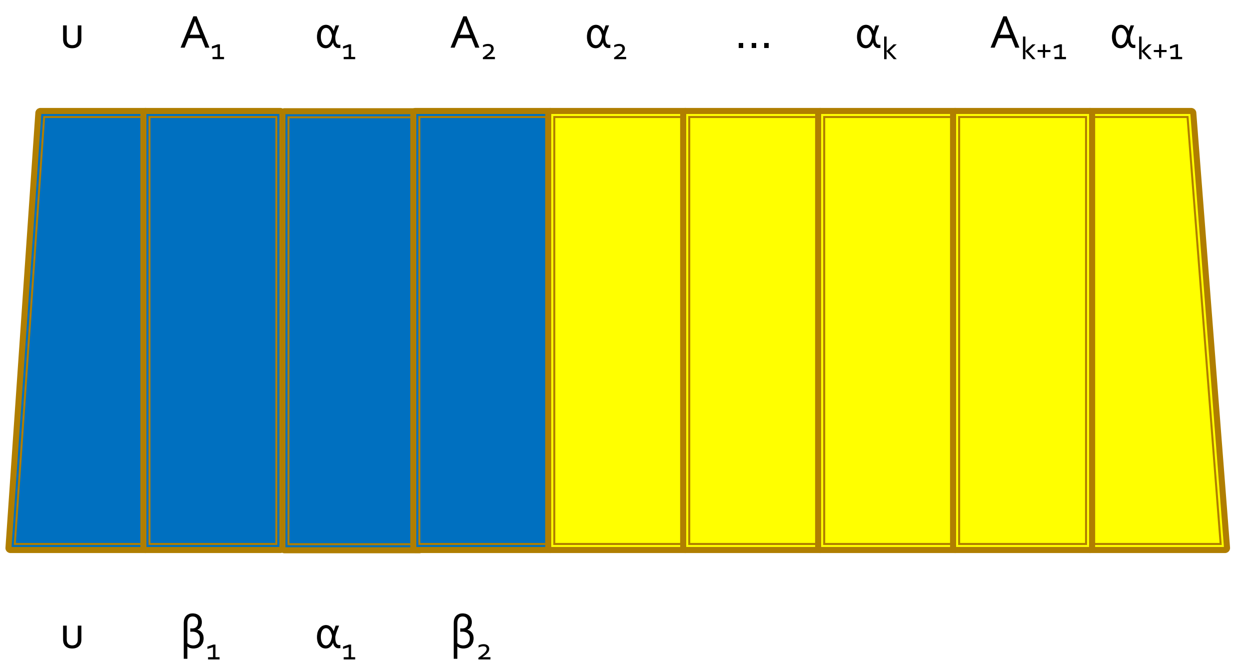
Delayed Derivation - Example



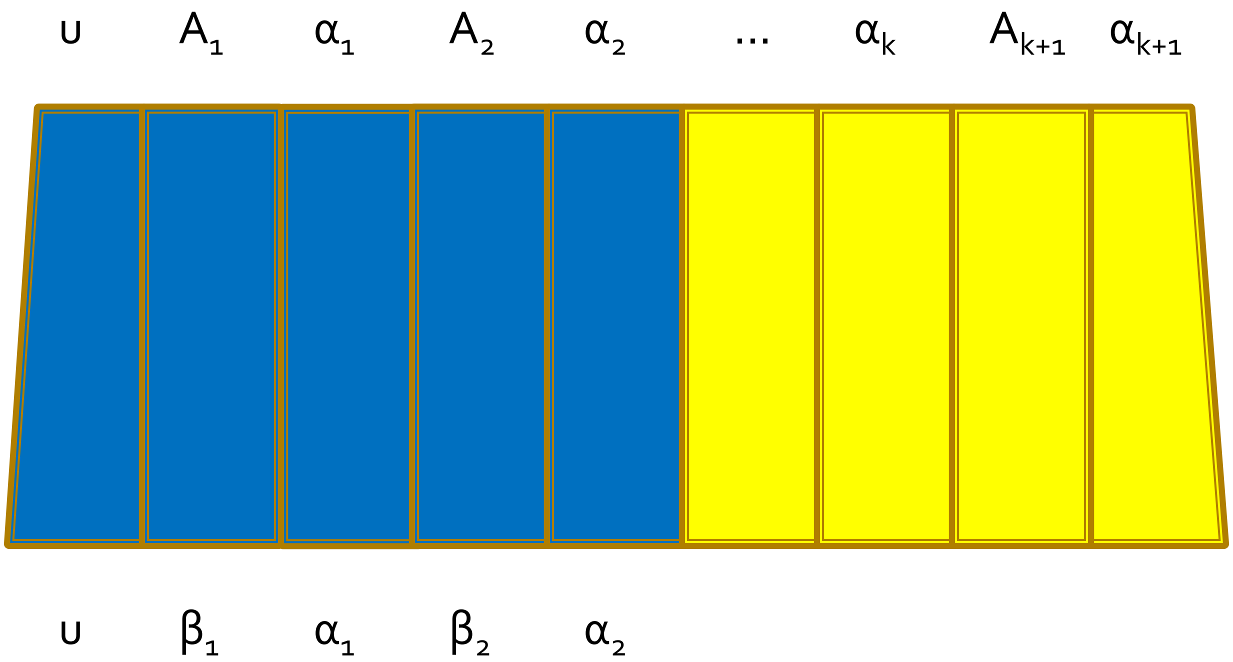
Delayed Derivation - Example



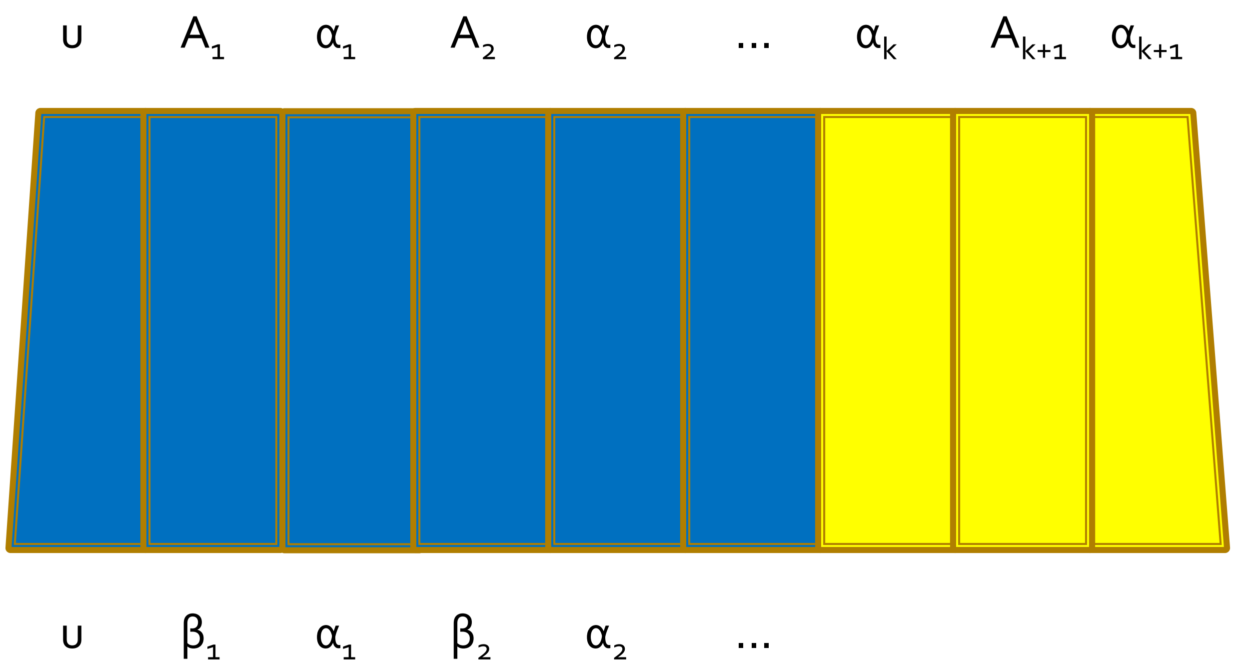
Delayed Derivation - Example



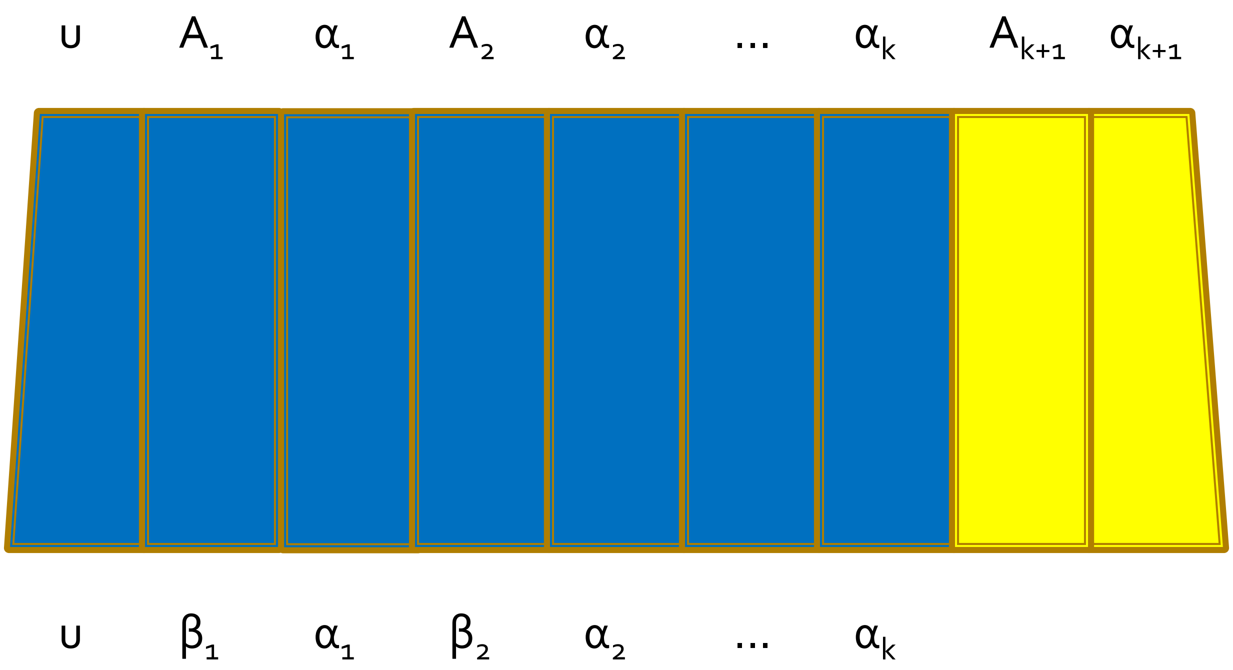
Delayed Derivation - Example



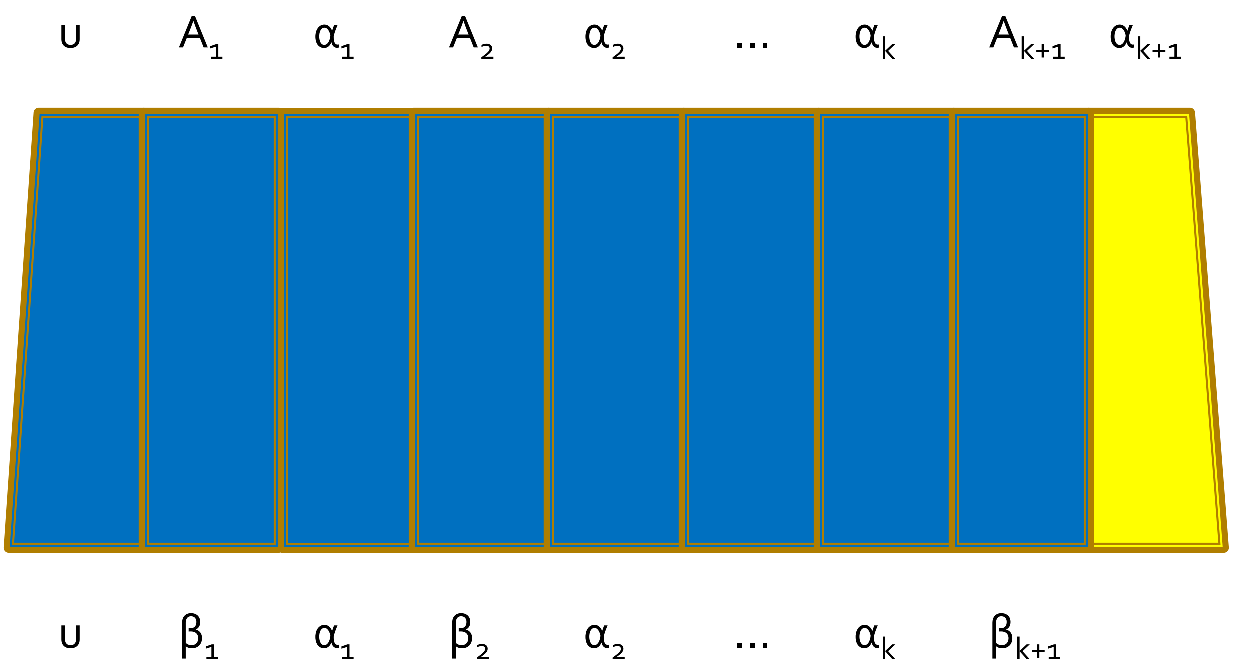
Delayed Derivation - Example



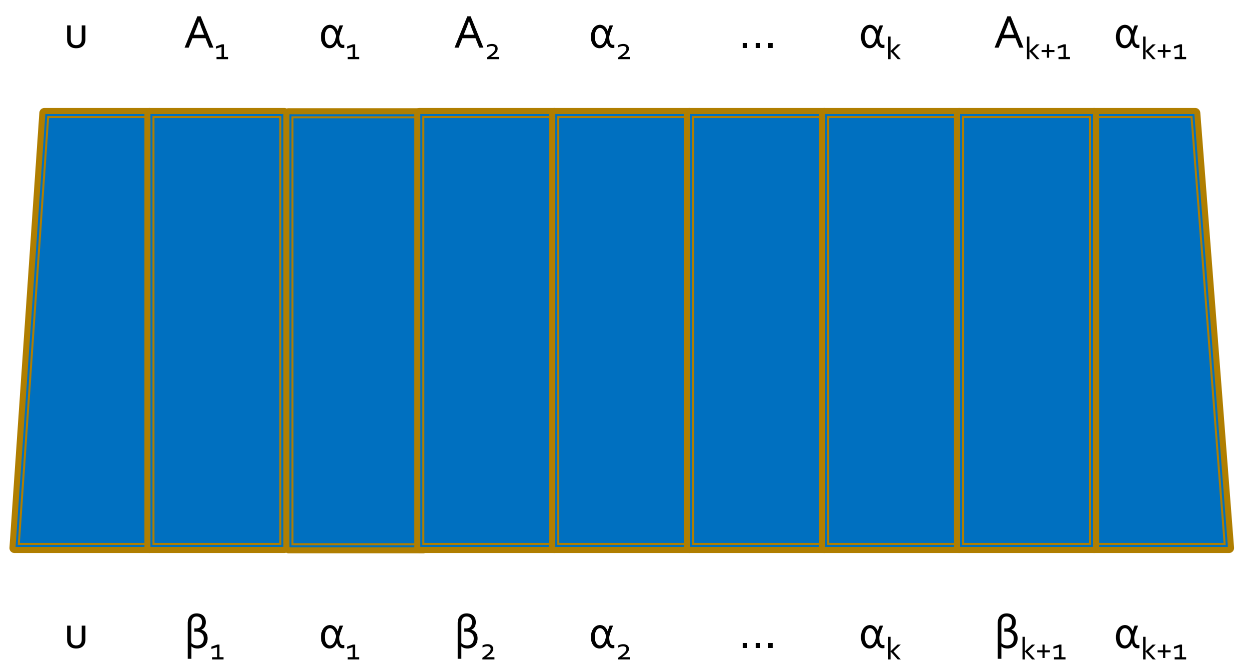
Delayed Derivation - Example



Delayed Derivation - Example



Delayed Derivation - Example



Lemma 1

- $g(1, uA_1\alpha_1 \ldots \alpha_k A_{k+1}\alpha_{k+1}) =$
  $h(uA_1\alpha_1 \ldots \alpha_k A_{k+1}\alpha_{k+1}), i \in \{1, \ldots, k\}$
- for some
  $k \in N$,
  $u \in T^*$,
  $x_i \in (V \setminus \{A_{i+1}\})^*$,
  $(A_1, \ldots, A_{k+1}) \longrightarrow (\beta_1, \ldots, \beta_{k+1}) \in P$

Lemma 2

- $\alpha \in V^*$ for some $k \in N$
- $g_k(1, \ldots g_2(1, g_1(1, \alpha))) \quad = h_k(\ldots h_2(h_1(\alpha))),$

Lemma 2

- $\alpha \in V^*$ for some $k \in N$
- $g_k(1, \ldots g_2(1, g_1(1, \alpha))) = h_k(\ldots h_2(h_1(\alpha)))$
- $g_k(1, \ldots g_2(1, \alpha_1))) = h_k(\ldots h_2(\alpha_1))$

Lemma 2

- $\alpha \in V^*$ for some $k \in N$
- $g_k(1, \ldots g_2(1, g_1(1, \alpha))) = h_k(\ldots h_2(h_1(\alpha)))$
- $g_k(1, \ldots g_3(g_2(1, \alpha_1))) = h_k(\ldots h_3(h_2(\alpha_1)))$
- $g_k(1, \ldots g_3(1, \alpha_2))) = h_k(\ldots h_3(\alpha_2))$

Lemma 2

- $\alpha \in V^*$ for some $k \in N$
- $g_k(1, \ldots g_2(1, g_1(1, \alpha))) = h_k(\ldots h_2(h_1(\alpha)))$
- $g_k(1, \ldots g_3(g_2(1, \alpha_1))) = h_k(\ldots h_3(h_2(\alpha_1)))$
- $g_k(1, \ldots g_3(1, \alpha_2)) = h_k(\ldots h_3(\alpha_2))$
- $g_k(1, \alpha_{k-1}) = h_k(\alpha_{k-1})$

Lemma 2

- $\alpha \in V^*$ for some $k \in N$
- $g_k(1, \ldots g_2(1, g_1(1, \alpha)))$ $= h_k(\ldots h_2(h_1(\alpha)))$
- $g_k(1, \ldots g_3(g_2(1, \alpha_1)))$ $= h_k(\ldots h_3(h_2(\alpha_1)))$
- $g_k(1, \ldots g_3(1, \alpha_2))$ $= h_k(\ldots h_3(\alpha_2))$
- $g_k(1, \alpha_{k-1})$ $= h_k(\alpha_{k-1})$
- α_k $= \alpha_k$

Lemma 3

- Lazy evaluation of $g_{k+1}(1, \ldots g_1(1, \omega_1 \ldots \omega_m)) = h_{k+1}(\ldots h_1(\omega_1 \ldots \omega_m)),$
- $\omega_i \in V$, $i \in \{1, \ldots, m\}$ for some $m \geq 1$ and $k \geq 1$.

Lemma 3 – Regular Derivation

- $\omega_i \in V$, $i \in \{1, \dots, m\}$ for some $m \geq 1$ and $k \geq 1$.
- $h_{k+1}(\dots h_2(h_1(\omega_1 \dots \omega_m)))$

Lemma 3 – Regular Derivation

- $\omega_i \in V$, $i \in \{1, . . . , m\}$ for some $m \geq 1$ and $k \geq 1$.
- $h_{k+1}(. . . h_2(h_1(\omega_1 . . . \omega_m))) =$
- $h_{k+1}(. . . h_3(h_2(\omega'_1 . . . \omega'_m)))$

Lemma 3 – Regular Derivation

- $\omega_i \in V$, $i \in \{1, \dots, m\}$ for some $m \geq 1$ and $k \geq 1$.
- $h_{k+1}(\dots h_2(h_1(\omega_1 \dots \omega_m))) =$
- $h_{k+1}(\dots h_3(h_2(\omega'_1 \dots \omega'_m))) =$
- $h_{k+1}(\dots h_3(\omega''_1 \dots \omega''_m))$

Lemma 3 – Regular Derivation

- $\omega_i \in V$, $i \in \{1, . . . , m\}$ for some $m \geq 1$ and $k \geq 1$.
- $h_{k+1}(. . . h_2(h_1(\omega_1 . . . \omega_m))) =$
- $h_{k+1}(. . . h_3(h_2(\omega'_1 . . . \omega'_m))) =$
- $h_{k+1}(. . . h_3(\omega''_1 . . . \omega''_m)) =$
- ...
- $h_{k+1}(\omega'''_1 . . . \omega'''_m)$

Lemma 3 – Regular Derivation

- $\omega_i \in V$, $i \in \{1, . . . , m\}$ for some $m \geq 1$ and $k \geq 1$.
- $h_{k+1}(. . . h_2(h_1(\omega_1 . . . \omega_m))) =$
- $h_{k+1}(. . . h_3(h_2(\omega'_1 . . . \omega'_m))) =$
- $h_{k+1}(. . . h_3(\omega''_1 . . . \omega''_m)) =$
- ...
- $h_{k+1}(\omega'''_1 . . . \omega'''_m) =$
- $\omega''''_1 . . . \omega''''_m$

Lemma 3 – Delayed Derivation

- $\omega_i \in V$, $i \in \{1, \ldots, m\}$ for some $m \geq 1$ and $k \geq 1$.
- $g_{k+1}(1, \ldots g_2(1, g_1(1, \omega_1 \ldots \omega_m)))$

Lemma 3 – Delayed Derivation

- $\omega_i \in V$, $i \in \{1, \dots, m\}$ for some $m \geq 1$ and $k \geq 1$.
- $g_{k+1}(1, \dots g_2(1, g_1(1, \omega_1 \dots \omega_m))) =$
- $g_{k+1}(1, \dots g_2(1, \omega'_1\, g_1(1', \omega_2 \dots \omega_m)))$

Lemma 3 – Delayed Derivation

- $\omega_i \in V$, $i \in \{1, . . . ,m\}$ for some $m \geq 1$ and $k \geq 1$.
- $g_{k+1}(1, . . . g_2(1, g_1(1, \omega_1 . . . \omega_m))) =$
- $g_{k+1}(1, . . . g_2(1, \omega'_1 g_1(1', \omega_2 . . . \omega_m))) =$
- $g_{k+1}(1, . . . \omega''_1 g_2(1', g_1(1', \omega_2 . . . \omega_m)))$

Lemma 3 – Delayed Derivation

- $\omega_i \in V$, $i \in \{1, . . . , m\}$ for some $m \geq 1$ and $k \geq 1$.
- $g_{k+1}(1, . . . g_2(1, g_1(1, \omega_1 . . . \omega_m))) =$
- $g_{k+1}(1, . . . g_2(1, \omega'_1 g_1(1', \omega_2 . . . \omega_m))) =$
- $g_{k+1}(1, . . . \omega''_1 g_2(1', g_1(1', \omega_2 . . . \omega_m))) =$
- ...
- $g_{k+1}(1, \omega'''_1 g_k(1', . . . g_2(1', g_1(1', \omega_2 ... \omega_m))))$

Lemma 3 – Delayed Derivation

- $\omega_i \in V$, $i \in \{1, . . . , m\}$ for some $m \geq 1$ and $k \geq 1$.
- $g_{k+1}(1, . . . g_2(1, g_1(1, \omega_1 . . . \omega_m))) =$
- $g_{k+1}(1, . . . g_2(1, \omega'_1\, g_1(1', \omega_2 . . . \omega_m))) =$
- $g_{k+1}(1, . . . \omega''_1\, g_2(1', g_1(1', \omega_2 . . . \omega_m))) =$
- ...
- $g_{k+1}(1, \omega'''_1\, g_k(1', . . . g_2(1', g_1(1', \omega_2 ... \omega_m))))=$
- $\omega''''_1\, g_{k+1}(1', g_k(1', . . . g_2(1', g_1(1', \omega_2 ... \omega_m))))$

Lemma 3 – Delayed Derivation

- $\omega_i \in V$, $i \in \{1, . . . ,m\}$ for some $m \geq 1$ and $k \geq 1$.
- $g_{k+1}(1, . . . g_2(1, g_1(1, \omega_1 . . . \omega_m))) =$
- $g_{k+1}(1, . . . g_2(1, \omega'_1 g_1(1', \omega_2 . . . \omega_m))) =$
- $g_{k+1}(1, . . . \omega''_1 g_2(1', g_1(1', \omega_2 . . . \omega_m))) =$
- ...
- $g_{k+1}(1, \omega'''_1 g_k(1',. . . g_2(1', g_1(1', \omega_2 ...\omega_m))))=$
- $\omega''''_1 g_{k+1}(1', g_k(1',. . . g_2(1', g_1(1', \omega_2 ...\omega_m))))$
- ...
- $\omega''''_1 . . . \omega''''_m$

Theorem

- Lazy evaluation of
  $g_m(1, g_{m-1}(1, \ldots g_1(1, \omega_1 \ldots \omega_j))) =$
  $h_m(h_{m-1}(\ldots h_1(\omega_1 \ldots \omega_j))) = w$
  $\equiv$
  $\omega_1 \ldots \omega_j \Rightarrow^m w$

- $w, \omega_i \in V$, $i \in \{1, \ldots, j\}$ for some $j, m \in N$.

Conclusion

- Usage of function instead of derivation steps.
- Lazy evaluation of delayed execution of SCG has been presented.
- Equivalency of lazy evaluated delayed executed function and the function representing leftmost derivation over a sentential form.

Open Questions

- Compilers using delayed execution of productions.
- To compare with existing implementations.

Thank you for your attention