

Chan and Vese Level Set Method with Modeling the Brightness of Background

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November 16, 2008

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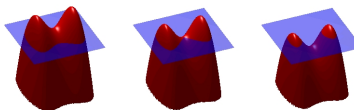
The Problem Task

Variational methods in image processing

- image segmentation with an active contour
- active contour is defined by control points
- evolving contour can, for example, split up into more parts
- the problem is to mathematically express these "wild" states

Level Set

- the problem of active contour description can be superseded by introduction of a level set
- define a function over an image $\phi : \mathbb{R}^2 \rightarrow \mathbb{R}$
- in the image plane (xy) is $\phi = 0$ and forms a contour (zero level set)
- problematic definition of the active contour is overcome, all "wild" states happen in the ϕ



Chan-Vese Method

- works for images with constant brightness value of foreground and background
- model is based on minimizing the functional:

Original functional

$$F(c_o, c_b, C) = \mu \cdot \text{Length}(C) + \nu \cdot \text{Area}(\text{inside}(C)) + \lambda_o \int_{\text{inside}(C)} (u(x) - c_o)^2 dx + \lambda_b \int_{\text{outside}(C)} (u(x) - c_b)^2 dx$$

- C - contour
- u - image brightness value
- c_o - average value of the brightness inside the contour
- c_b - average value of the brightness outside the contour

Level Set Function Introduction

- to minimize the functional, a function $\phi(x)$, $x \in \Omega$ is introduced
- $\phi > 0$ – inside the objects
- $\phi = 0$ – on their boundaries
- $\phi < 0$ – outside the objects

Heaviside function is used to rewrite the functional into level set formulation.

Heaviside function

$$H(z) = \begin{cases} 1, & \text{if } z \geq 0 \\ 0, & \text{if } z < 0 \end{cases}, \delta(z) = \frac{d}{dz} H(z)$$

Level Set Formulation

Functional with the level set

$$F(c_o, c_b, \phi) = \mu \int_{\Omega} |\nabla H(\phi)| dx + \nu \int_{\Omega} H(\phi) dx + \\ + \lambda_o \int_{\Omega} (u - c_o)^2 H(\phi) dx + \lambda_b \int_{\Omega} (u - c_b)^2 (1 - H(\phi)) dx$$

Average brightnesses over objects and background

$$c_o(\phi) = \frac{\int_{\Omega} u H(\phi) dx}{\int_{\Omega} H(\phi) dx}, \quad c_b(\phi) = \frac{\int_{\Omega} u (1 - H(\phi)) dx}{\int_{\Omega} (1 - H(\phi)) dx}$$

Minimizing the Functional

Corresponding Euler-Lagrange equation

$$\delta(\phi) \left[\mu \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu - \lambda_o (u - c_o)^2 + \lambda_b (u - c_b)^2 \right] = 0 \text{ in } \Omega$$

Introducing $\phi(t, x)$ by parameterizing the descent direction by time $t \geq 0$, and $\phi(0, x) = \phi_0(x)$ (chosen initial contour), we get:

System for solving ϕ

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= \delta(\phi) \left[\mu \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu - \lambda_o (u - c_o)^2 + \lambda_b (u - c_b)^2 \right] \text{ in } \Omega, \\ \phi(0, x) &= \phi_0(x) \text{ in } \Omega \end{aligned}$$

Chan-Vese Image Segmentation Example



Background Brightness Term

- the term $\lambda_b \int_{\Omega} (u - c_b)^2 (1 - H(\phi)) dx$ minimizes the functional regarding to the background brightness
- only images with constant background brightness can be segmented

Consider the background being a linear function

- we take a plane as a representative
- we can describe images with background brightness that increases or decreases along the x or y axis

New background term

$$\lambda_b \int_{\Omega} (u - c_b - ar - bs)^2 (1 - H(\phi)) dx$$

- r, s – image coordinates
- c_b – constant shift of the brightness

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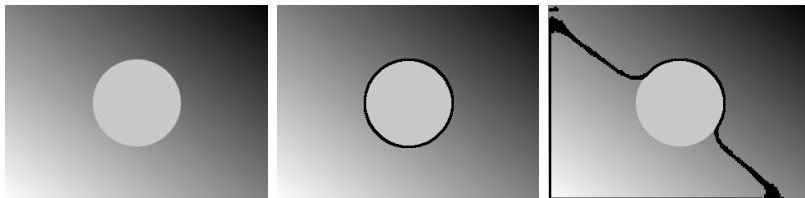
Euler-Lagrange Equation for the New Functional

Corresponding Euler-Lagrange equation

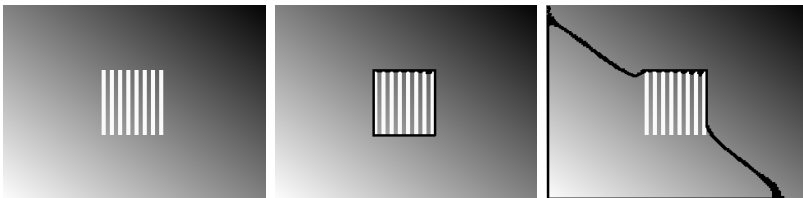
$$\delta(\phi) \left[\mu \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu - \lambda_o (u - c_o)^2 + \lambda_b (u - c_b - ar - bs)^2 \right] = 0$$

- c_b, a, b – derived from the equation system of the new functional

Artificial Image I



Artificial Image II



Real Life Image



Summary

- ① we introduced a modification to the original Chan-Vese method
- ② images with background being a linear function can be segmented
- ③ objects are however of constant brightness

Thank You